Geometric Maxwell-Lorentz : Discrete e⁻, p⁺

$$\{\hat{E},\hat{B}\} = \{E,B\}/e \sim [l^{-2}]$$

$$\nabla\cdot\hat{E}(x) = 4\pi\sum_{j=1}^{N} s_j\,\delta(x-x_j) \qquad s_j = \pm 1$$

$$\nabla\cdot\hat{B}(x) = 0$$

photons

$$c\nabla\times\hat{E} = -\dot{\hat{B}}$$

$$c\nabla\times\hat{B} = \dot{\hat{E}} + 4\pi\sum_{j=1}^{N} s_j\,\mathrm{v}_j\,\delta(x-x_j)$$

$$a_k = e^2\frac{s_k}{m_k}\left[\hat{E} + \left(\frac{\mathrm{v}_k}{c}\right)\times\hat{B}\right]$$

$$\mathcal{E} = \frac{e^2}{8\pi}\left(\hat{E}^2+\hat{B}^2\right)$$

Fundamental Constants

t, l | c

m | m_e

$$e^2 = 1.44\ \mathrm{eV\cdot nm}$$

$$hc = 1240.\ \mathrm{eV\cdot nm}$$

$$r_e \equiv \frac{e^2}{m_e c^2} = 2.82\text{x}10^{-15}\ \mathrm{m}$$

or

$$\lambda\!\!\!^{-}_c \equiv \frac{\hbar c}{m_e c^2} = 386.\text{x}10^{-15}\ \mathrm{m}$$

$$a_0 = \lambda\!\!\!^{-2}_c / r_e \qquad r_e/\lambda\!\!\!^{-}_c = 1/137.$$

$$\hat{\mu}_B = \lambda\!\!\!^{-}_c/2 \qquad \frac{G m_p^2}{e^2} = 0.81\text{x}10^{-36}$$

Plasma Parameters :

Size scale r_e , n

Rate scale $\beta \equiv \bar{\mathrm{v}}/c$

$$\bar{\mathrm{v}}^2 \equiv T/m_e$$

$$\Omega_{ce} \equiv eB/mc = r_e\,\hat{B}\,c$$

$$\omega_p^2 \equiv 4\pi n e^2/m = n\,r_e\,4\pi c^2$$

$$\lambda_D^{-2} \equiv 4\pi n e^2/T = n\,r_e\,\beta^{-2}\,4\pi$$

$$\text{"}\beta_p^{-1}\text{"} \equiv \frac{B^2}{8\pi nT} = \frac{r_e\,\hat{B}^2\,\beta^{-2}}{n\ 8\pi}$$

$$V_A^2 \equiv \frac{B^2}{4\pi n\,m_p} = \frac{r_e\,\hat{B}^2}{n}\,\frac{c^2\,m_e}{4\pi\,m_p}$$

Fluidize : $N \to N/\varepsilon$

$\varepsilon \to 0$

$$\{\hat{E},\hat{B}\} \to \{\hat{E},\hat{B}\}/\varepsilon$$

$$e \to \varepsilon e$$

$$m \to \varepsilon m$$

$$T \to \varepsilon T$$

$$r_e \to \varepsilon r_e \to 0$$

Constant :

{E, B}

Q, $\mathcal{E}$

M, $\bar{\mathrm{v}}$

nT

MHD :

$$Q = 0$$

$$E = 0$$

$$\nabla\cdot J = 0$$

Fluidize: $\to 0$

$$b \equiv e^2/T = r_e\,\beta^{-2} \to 0$$

$$\nu_c = n\,\bar{\mathrm{v}}\,\mathrm{b}^2\,ln\Lambda = n\,r_e^{\,2}\beta^{-3} c\,ln\Lambda \to 0$$

$$\rho_d = \frac{m_e}{e^2}\frac{\nu_c}{n} = r_e\beta^{-3}\,ln\Lambda/c \to 0$$

$$g \equiv \frac{1}{n\lambda_D^{\,3}} = [n r_e^{\,3}\beta^{-6}(4\pi)^3]^{1/2} \to 0$$

$\Gamma \propto g^{1/3}$ (correlation)

Geometric Maxwell-Lorentz : Discrete e^-, p^+

In geometric units, $\{\hat{E},\hat{B}\}$ have dimension length^{-2} .
The fundamental constant e^2 = 1.44 eV.nm sets the EM energy scale;
or equivalently, $r_e = e^2/m_e c^2 = 2.8 \times 10^{-15}$ m sets the (only) size scale .

The analogous quantum constant hc = 1240. eV.nm ,
or equivalently the Compton wavelength λ_c ,
then determine the Bohr radius a_0 and the Bohr magneton μ_B.

Plasma parameters are then determined by
by the size scale set by re, density n, and geometric fields $\{\hat{E},\hat{B}\}$,
and by the rate scale $\beta = \overline{v} / c$.

The fluid approximation “infinitely partitions” particles, keeping e/m constant;
but this makes $r_e = 0$, eliminating the only scale size in the equations, and
eliminating the physical basis for collisions, resistive dissipation, and correlation.
With separate e- and p+ fluids, dissipation can be re-inserted ad-hoc.

Magneto-Hydro Dynamics extends this mutilation of the Maxwell-Lorentz equations,
by eliminating all charge density, and all non-inductive electric fields,
retaining only a disembodied inductive (solenoidal, curling) current.