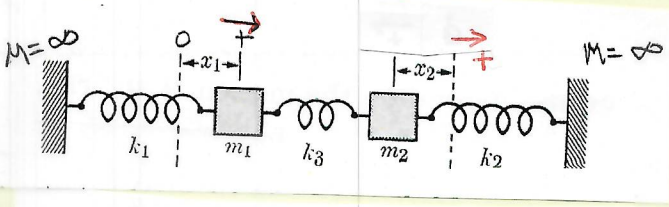
Symon, Mechanics (1960)

"Mingled Crossing" part 2

4 Feb 2022



$$m_1\ddot{x}_1 + b_1\dot{x}_1 + k_1'x_1 + k_3x_2 = 0, \qquad (4\text{–}180$$

$$m_2\ddot{x}_2 + b_2\dot{x}_2 + k_2'x_2 + k_3x_1 = 0, \qquad (4\text{–}181$$

$$k_1' = k_1 + k_3,$$

$$k_2' = k_2 + k_3.$$

Normal Modes:

$$x_1 = C_1e^{pt}, \qquad (4\text{–}141)$$

$$x_2 = C_2e^{pt}, \qquad (4\text{–}142)$$

phase-locked

where C_1, C_2 are constants. Note that the same time dependence is assumed for both x_1 and x_2, in order that the factor e^{pt} will cancel out when

$$\frac{C_2}{C_1} = + \frac{m_1p^2 + k_1'}{-k_3} = + \frac{-k_3}{m_2p^2 + k_2' + b_2p} \qquad (4\text{–}145)$$

b_1p +

$$m_1m_2p^4 + (m_2b_1 + m_1b_2)p^3 + (m_2k_1' + m_1k_2' + b_1b_2)p^2 + (b_1k_2' + b_2k_1')p + (k_1'k_2' - k_3^2) = 0. \qquad (4\text{–}182$$

This equation cannot be solved so easily as Eq. (4–147). The four roots for p are, in general, complex.

$$p \cong -\gamma_1 \pm i\omega_1,$$

$$p \cong -\gamma_2 \pm i\omega_2.$$

If two oscillators of different frequency are weakly coupled together, there are two normal modes of vibration of the system. In one mode, the oscillator of higher frequency oscillates at a frequency slightly higher than without coupling, and the other oscillates weakly out of phase at the same frequency. In the other mode, the oscillator of lowest frequency oscillates at a frequency slightly lower than without coupling, and the other oscillates weakly and in phase at the same frequency.

$\omega_{10} \neq \omega_{20}$

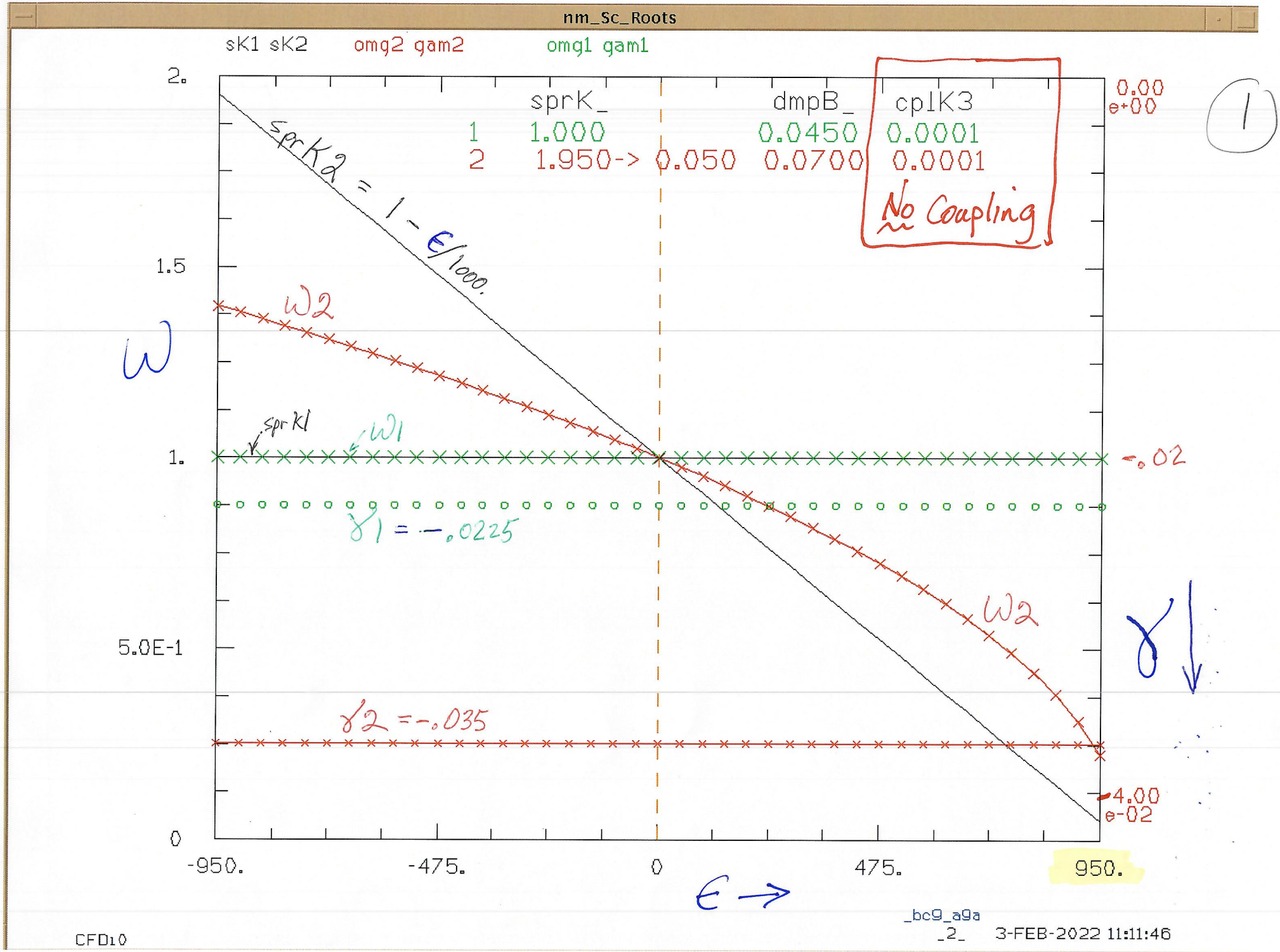
? ⇒ oscillator frequencies "push apart" due to spring coupling.

X Not a valid description with or without Damping.

"Avoiding Crossing"

NNP.ucsd.edu/Xfer/CFD_Symon_...2-9...PDF

...4-10 PDF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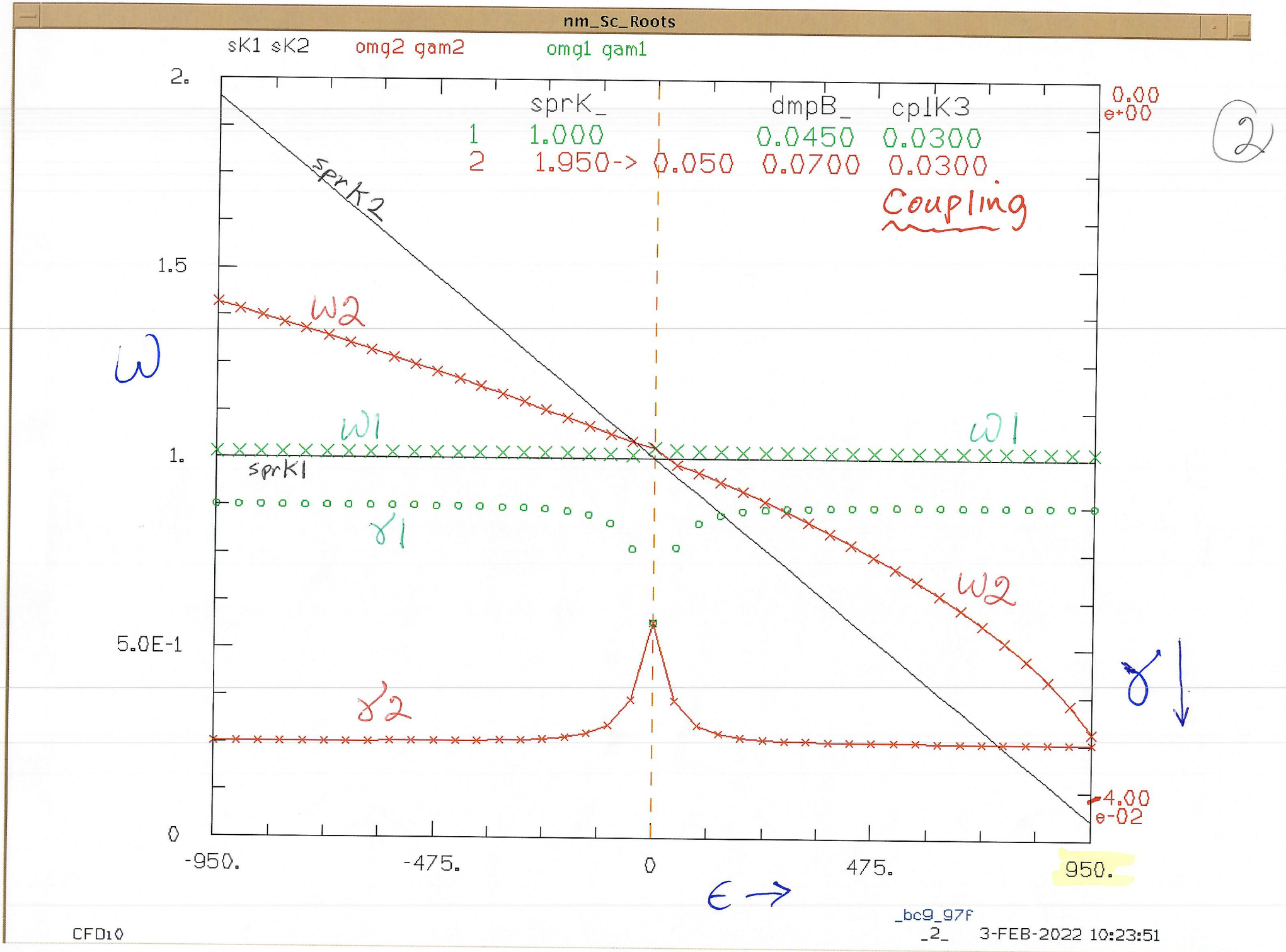
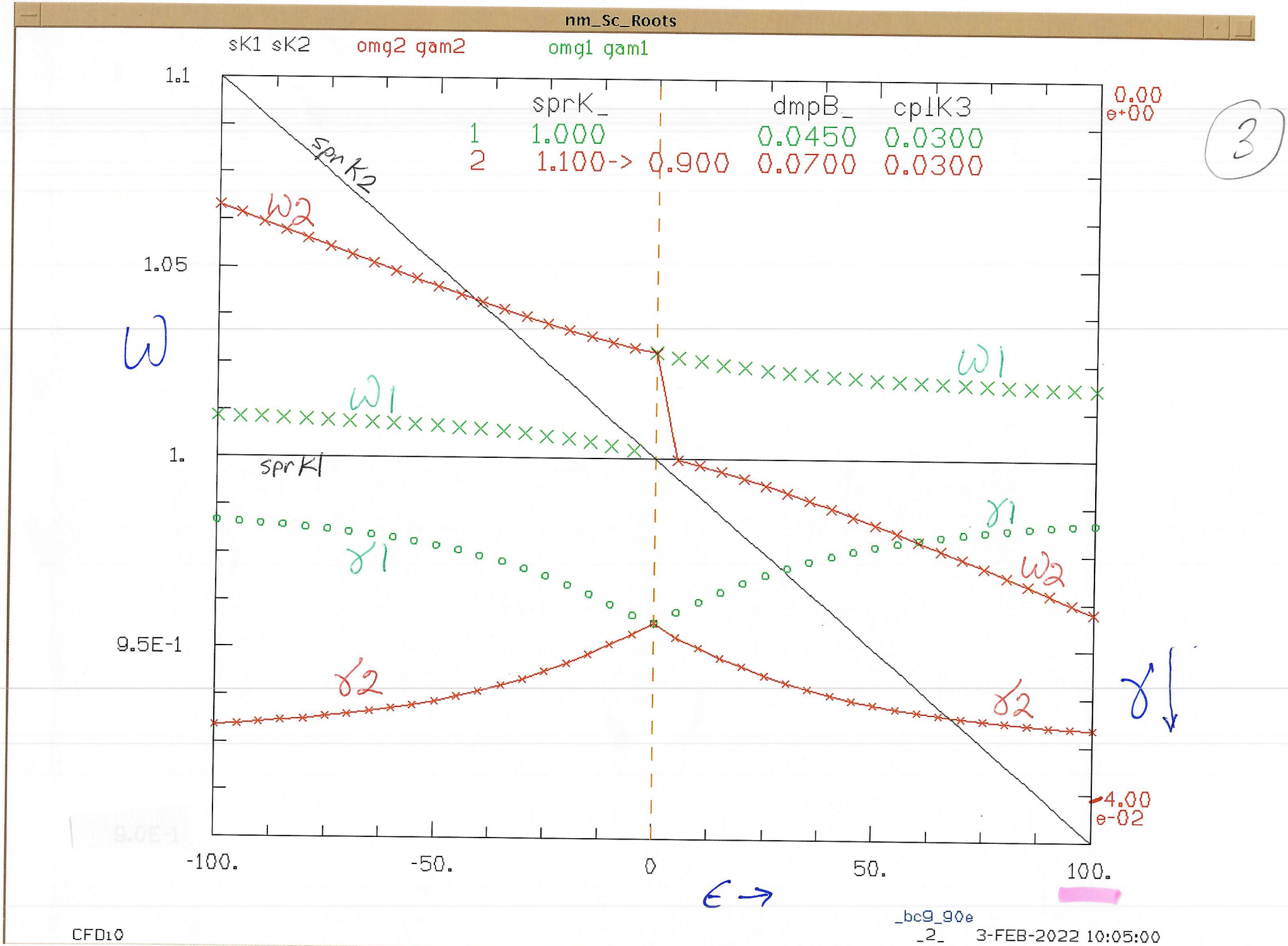
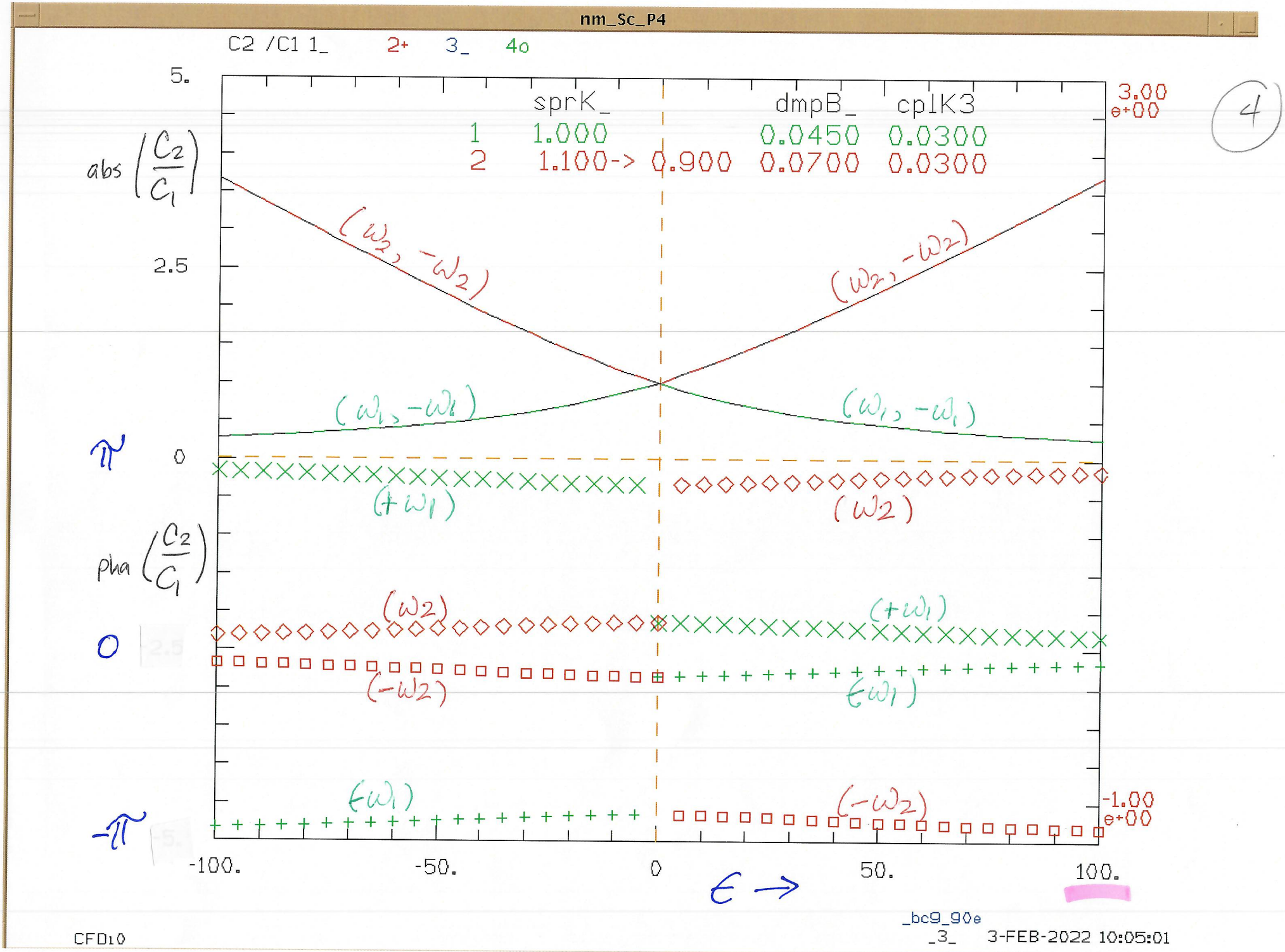
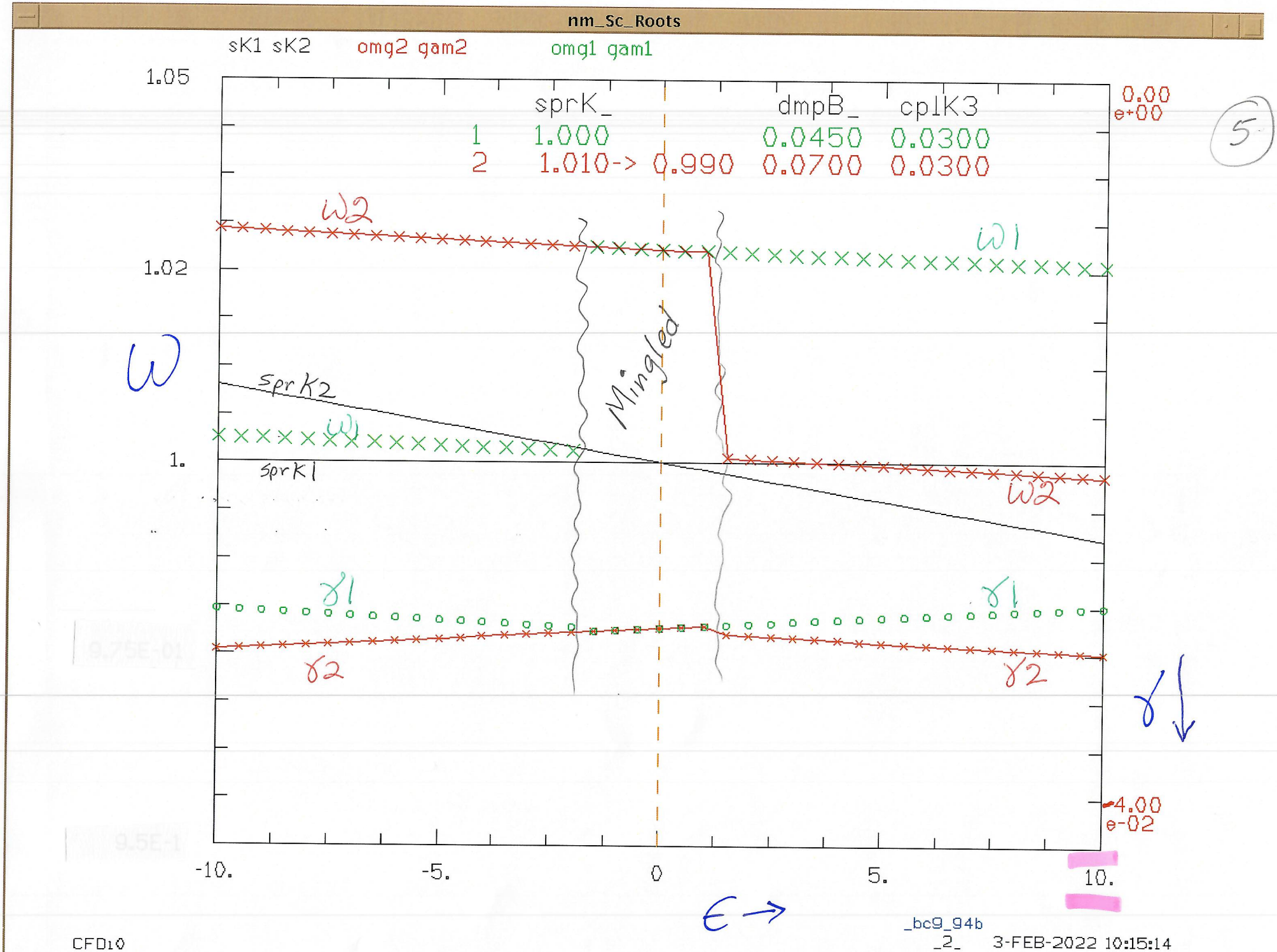
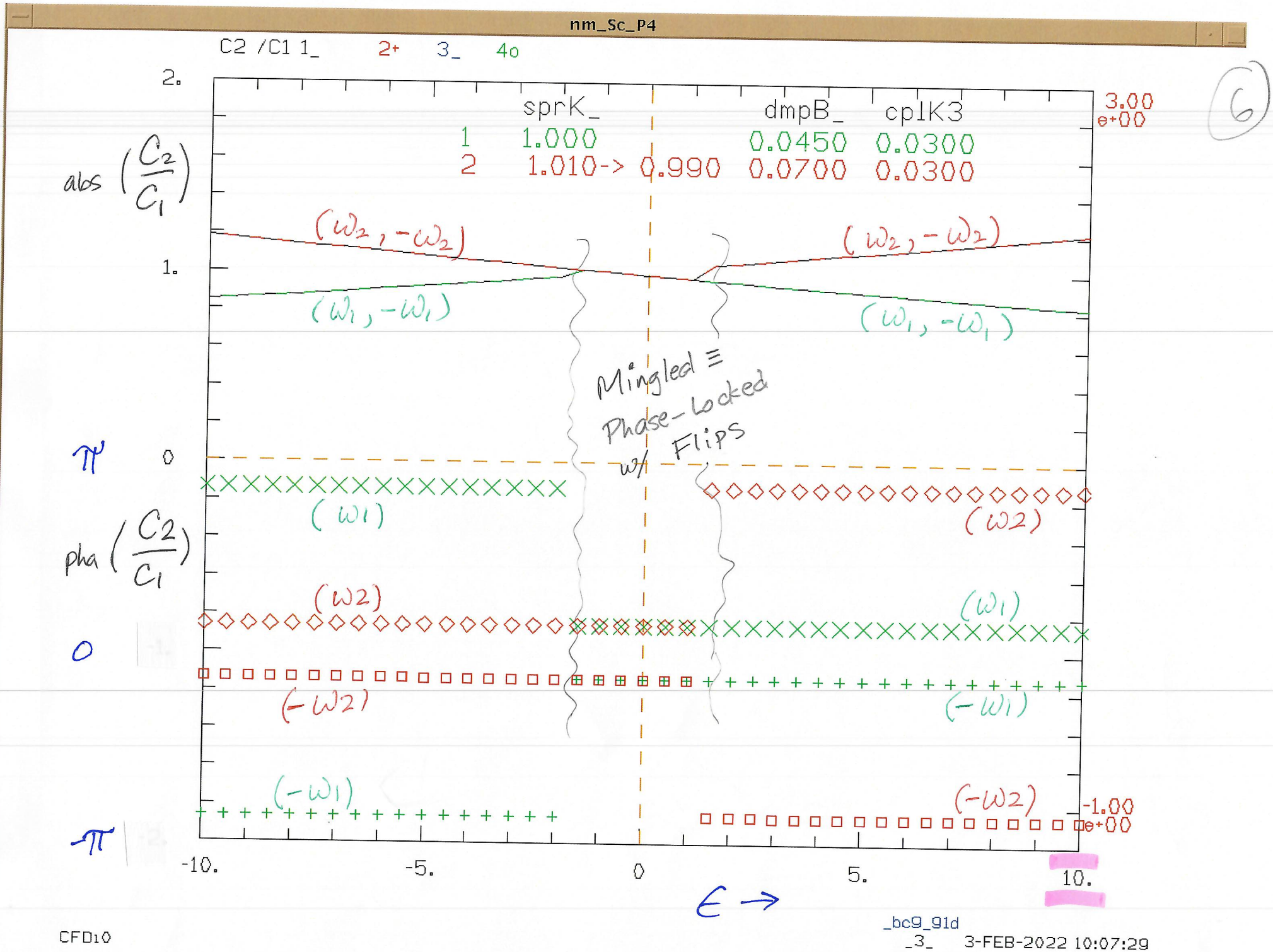
sK1 sK2
omg2 gam2
omg1 gam1
sprK_
dmpB_
cplK3
1 1.000 0.0450 0.0001
2 1.950-> 0.050 0.0700 0.0001
No Coupling
sprK2 = 1 - ε/1000.
W2
W1
sprK1
γ1 = -.0225
γ2 = -.035
W2
W
γ
ε →
2.
1.5
1.
5.0E-1
0
-950.
-475.
0
475.
950.
0.00 e+00
-.02
4.00 e-02
_bc9_a9a
_2_ 3-FEB-2022 11:11:46
CFD10


nm_Sc_Roots
sK1 sK2
omg2 gam2
omg1 gam1
sprK_ dmpB_ cplK3
1 1.000 0.0450 0.0300
2 1.950-> 0.050 0.0700 0.0300
Coupling
sprK2
sprK1
ω2
ω1
γ1
γ2
ω
γ
ε →
2.
1.5
1.
5.0E-1
0
0.00 e+00
-4.00 e-02
-950.
-475.
0
475.
950.
CFD10
_bc9_97f
_2_ 3-FEB-2022 10:23:51


nm_Sc_Roots
sK1 sK2
omg2 gam2
omg1 gam1
sprK_ dmpB_ cplK3
1 1.000 0.0450 0.0300
2 1.100-> 0.900 0.0700 0.0300
sprK2
sprK1
ω2
ω1
ω1
γ1
γ1
ω2
γ2
γ2
ω
γ↓
ε→
1.1
1.05
1.
9.5E-1
0.00 e+00
4.00 e-02
-100.
-50.
0
50.
100.
CFD10
_bc9_90e
_2_ 3-FEB-2022 10:05:00
3


nm_Sc_P4
C2 /C1 1_ 2+ 3_ 4o
5.
2.5
0
-π
sprK_ dmpB_ cplK3
1 1.000 0.0450 0.0300
2 1.100-> 0.900 0.0700 0.0300
3.00 e+00
-1.00 e+00
abs (C2/C1)
pha (C2/C1)
(ω2, -ω2)
(ω1, -ω1)
(+ω1)
(ω2)
(-ω2)
(-ω1)
-100.
-50.
0
50.
100.
ε →
_bc9_90e
_3_ 3-FEB-2022 10:05:01
CFD10


nm_Sc_Roots
sK1 sK2
omg2 gam2
omg1 gam1
sprK_ dmpB_ cplK3
1 1.000 0.0450 0.0300
2 1.010-> 0.990 0.0700 0.0300
1.05
1.02
1.
0.00
e+00
-4.00
e-02
-10.
-5.
0
5.
10.
ω2
ω1
sprK2
sprK1
Mingled
γ1
γ2
ω
γ
ϵ →
5
CFD10
_bc9_94b
_2_ 3-FEB-2022 10:15:14


nm_Sc_P4
C2 /C1 1_ 2+ 3_ 4o
sprK_ dmpB_ cplK3
1 1.000 0.0450 0.0300
2 1.010-> 0.990 0.0700 0.0300
3.00 e+00
-1.00 e+00
2.
1.
0
-10.
-5.
0
5.
10.
abs (C2/C1)
pha (C2/C1)
π
0
-π
(ω2, -ω2)
(ω1, -ω1)
Mingled ≡ Phase-Locked w/ Flips
(ω1)
(ω2)
(-ω2)
(-ω1)
ε →
CFD10
_bc9_91d
_3_ 3-FEB-2022 10:07:29