

Symon, Mechanics (1960)

"Mingled Crossing"
part 3

15-Feb-2022

$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k'_1 x_1 + k_3 x_2 = 0, \quad (4-180)$$

$$k'_1 = k_1 + k_3,$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k'_2 x_2 + k_3 x_1 = 0, \quad (4-181)$$

$$k'_2 = k_2 + k_3.$$

Normal Modes:

$$x_1 = C_1 e^{pt}, \quad (4-141)$$

$$x_2 = C_2 e^{pt}, \quad (4-142)$$

phase-locked

where C_1, C_2 are constants. Note that the same time dependence is assumed for both x_1 and x_2 , in order that the factor e^{pt} will cancel out when

$$\frac{C_2}{C_1} = + \frac{b_1 p + k'_1}{-k_3} = + \frac{-k_3}{m_2 p^2 + k'_2 + b_2 p} \quad (4-145)$$

$$m_1 m_2 p^4 + (m_2 b_1 + m_1 b_2) p^3 + (m_2 k'_1 + m_1 k'_2 + b_1 b_2) p^2 + (b_1 k'_2 + b_2 k'_1) p + (k'_1 k'_2 - k_3^2) = 0. \quad (4-182)$$

This equation cannot be solved so easily as Eq. (4-147). The four roots for p are, in general, complex.

$$p \approx -\gamma_1 \pm i\omega_1,$$

$$p \approx -\gamma_2 \pm i\omega_2.$$

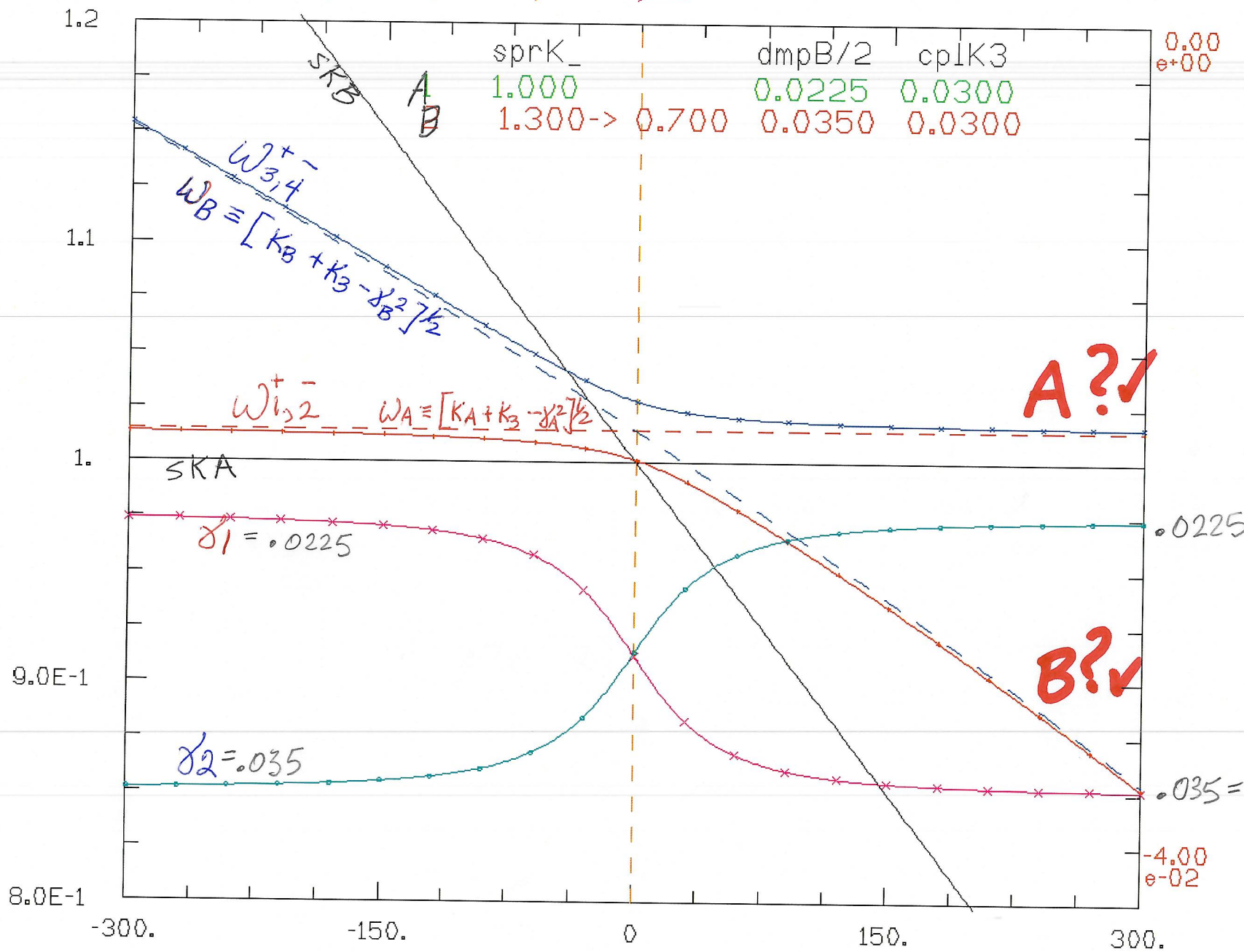
if two oscillators of different frequency are weakly coupled together, there are two normal modes of vibration of the system. In one mode, the oscillator of higher frequency oscillates at a frequency slightly higher than without coupling; and the other oscillates weakly out of phase at the same frequency. In the other mode, the oscillator of lowest frequency oscillates at a frequency slightly lower than without coupling, and the other oscillates weakly and in phase at the same frequency.

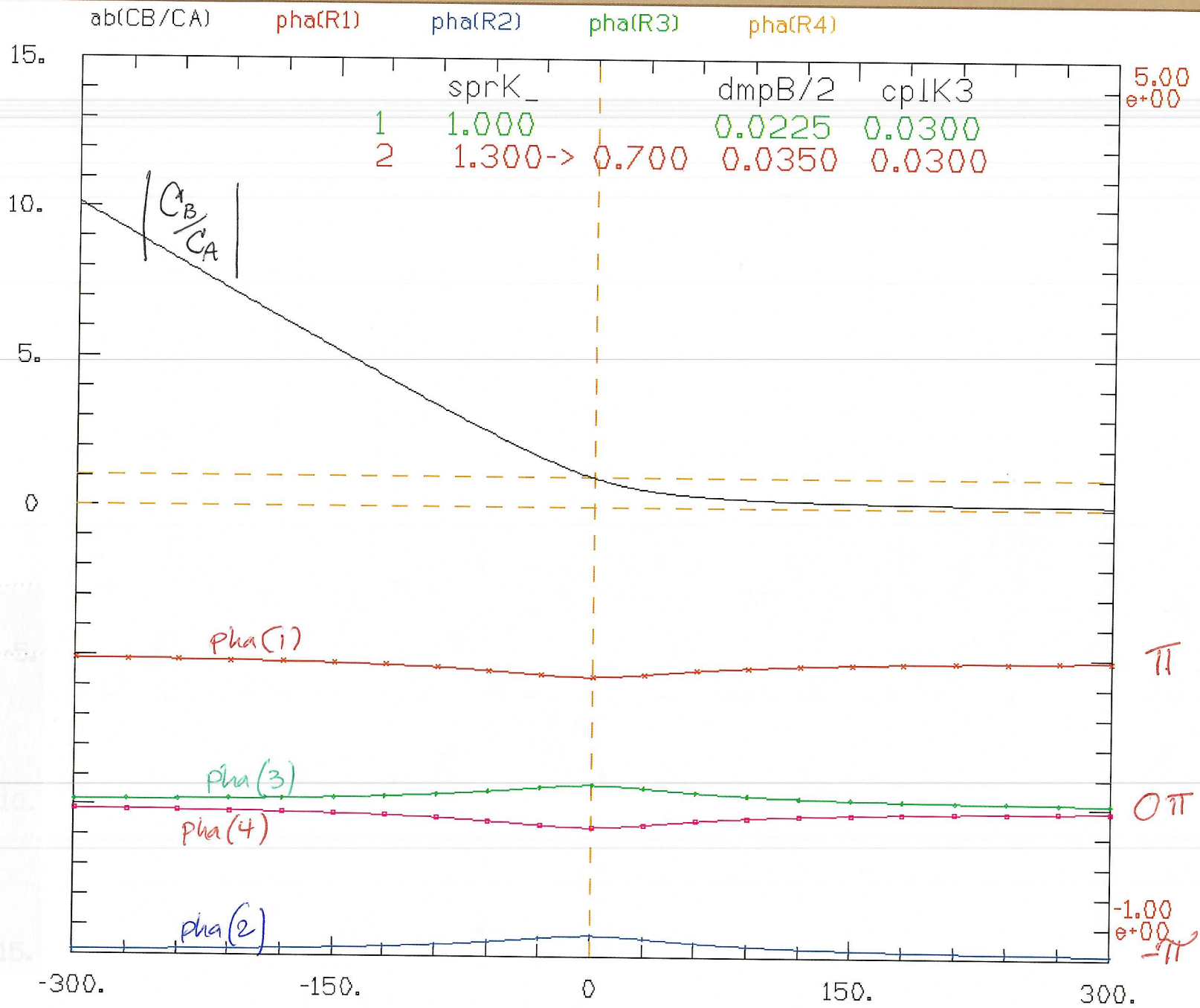
? \Rightarrow oscillator frequencies "push apart" due to spring coupling.
 X Not a valid description with or without Damping. "Avoiding Crossing"

sKA sKB omg1 Omg2 gam1 gam2

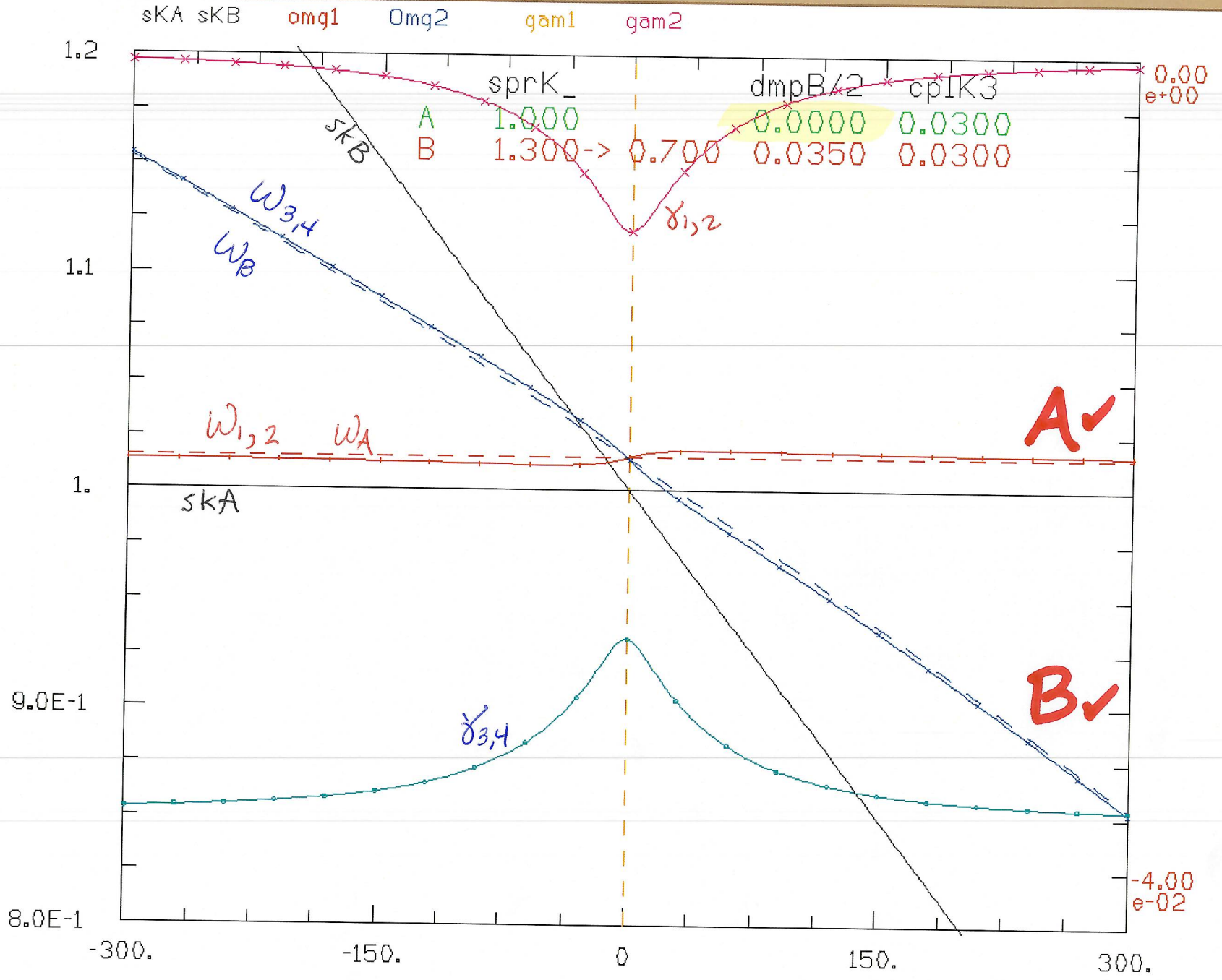
sprk_ dmpB/2 cplK3
 1.000 0.0225 0.0300
 1.300 -> 0.700 0.0350 0.0300

7

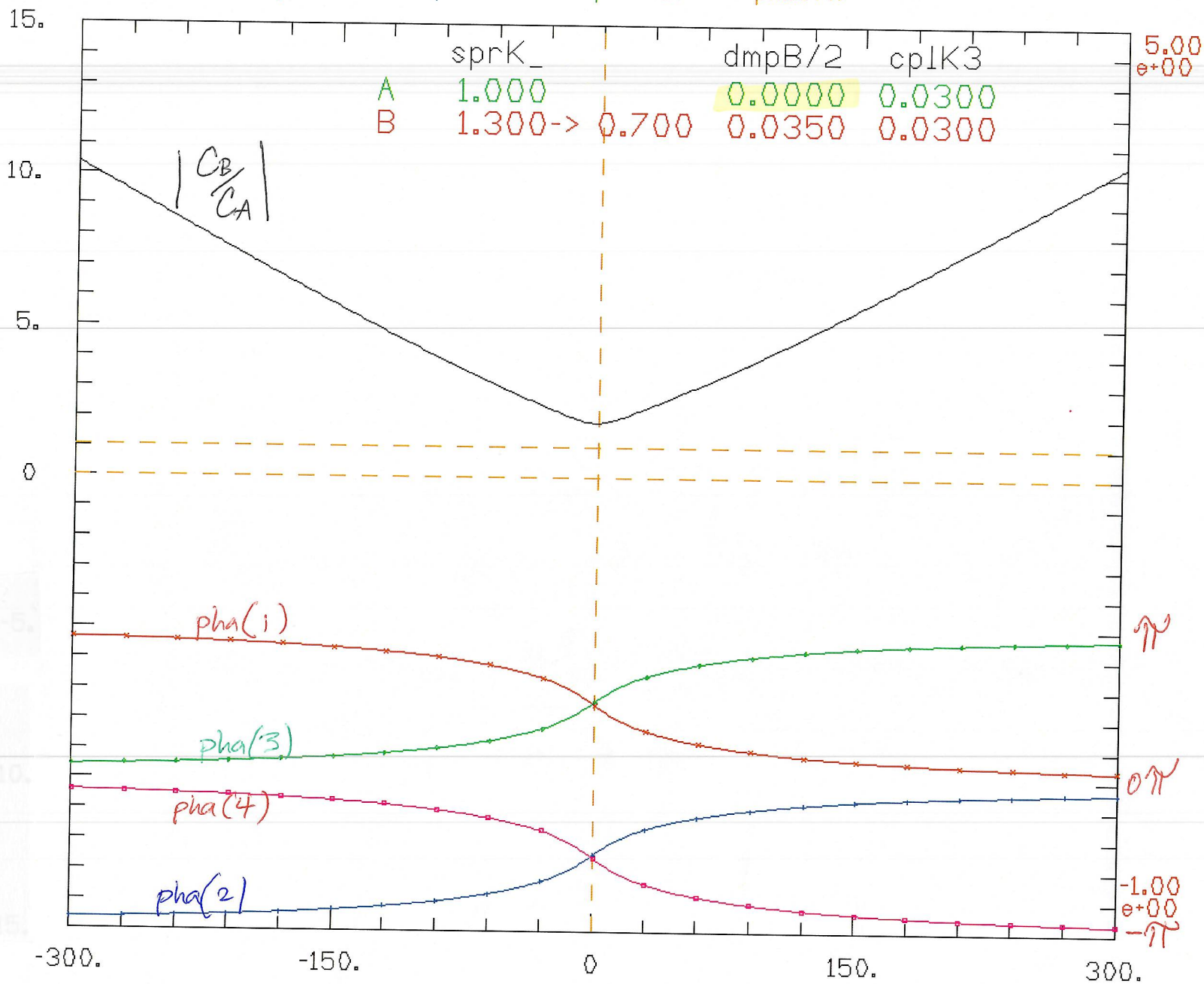




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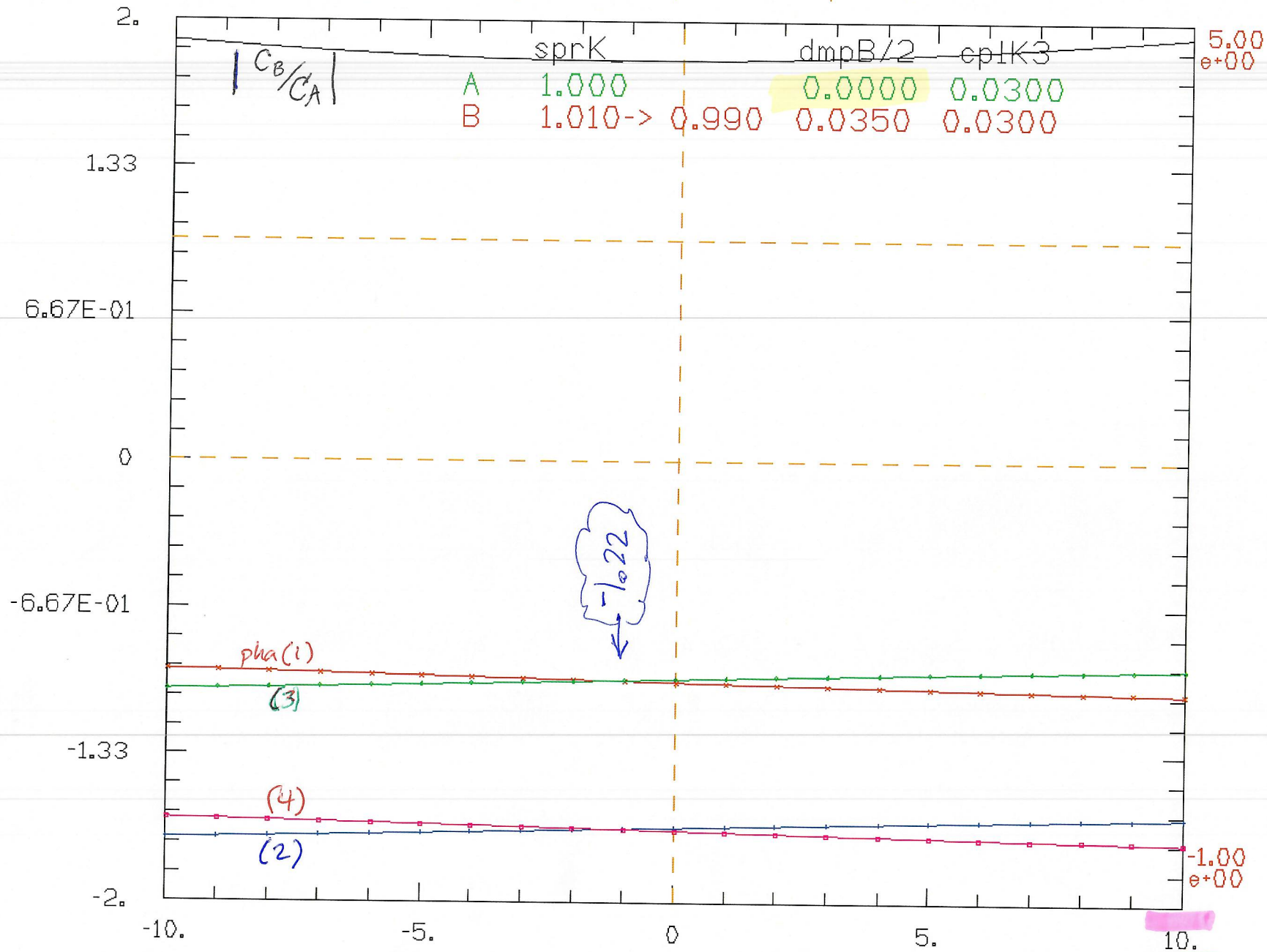


ab(CB/CA) pha(R1) pha(R2) pha(R3) pha(R4)



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ab(CB/CA) pha(R1) pha(R2) pha(R3) pha(R4)



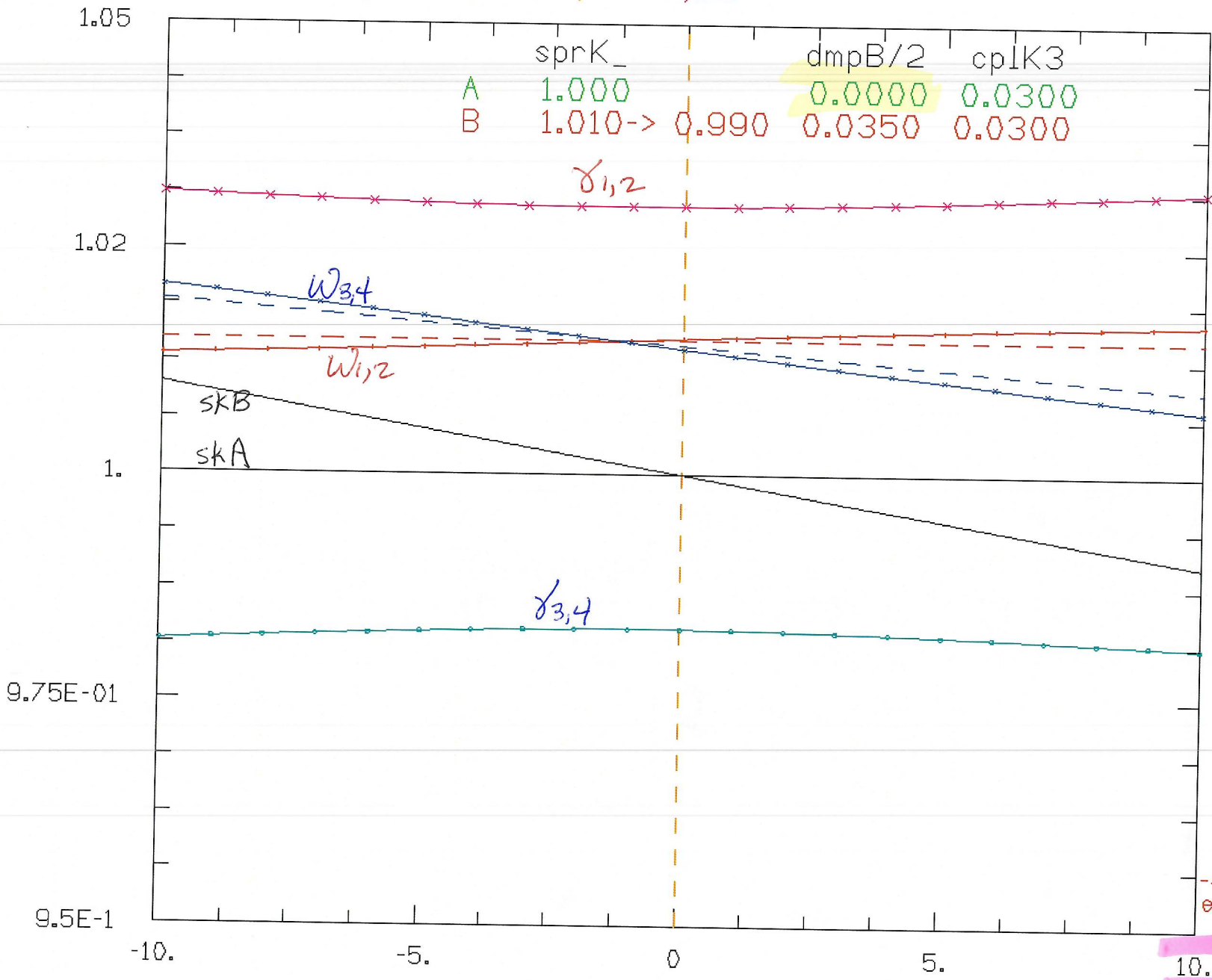
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sKA sKB omg1 Omg2 gam1 gam2

	sprK_	dmpB/2	cp1K3
A	1.000	0.0000	0.0300
B	1.010 -> 0.990	0.0350	0.0300

0.00
e+00

12



-4.00
e-02