Two Regimes of Asymmetry-Induced Transport in Non-neutral Plasmas

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Confinement of charged particles in cylindrical Penning-Malmberg traps depends strongly on cross-magnetic-field transport induced by electric and/or magnetic asymmetries. New measurements in pure-electron plasmas demonstrate two separate transport regimes depending on the particle bounce-to-rotation ratio, or rigidity, $\mathcal{R} \equiv \overline{f}_b/f_E$. For $\mathcal{R} < 10$, the transport scales as $V_a\mathcal{R}^{-2}$, where V_a is the strength of an applied electrostatic asymmetry. For $\mathcal{R} \gtrsim 10-20$, this "$\mathcal{R}^{-2}$ transport" ceases abruptly, leaving "B-independent" transport which scales as V_a^2 and does not depend directly on the rigidity.

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Single-species plasmas are readily confined with static electric and magnetic fields in cylindrical Penning-Malmberg traps [1]. These trapped plasmas are useful for studying basic plasma physics [2], such as collisional transport [3], two-dimensional vortex dynamics [4], and Coulomb crystals [5], as well as for technological applications such as pressure standards [6], ion-mass spectroscopy [7], frequency standards [8], and antimatter containment [9].

A fundamental consideration for many of these applications is the confinement time of the particles, which is often limited by asymmetry-induced transport. That is, static asymmetries due to imperfections in trap construction and alignment exert a drag (negative torque) on the rotating plasma, causing radial expansion and loss across the confining magnetic field. In contrast, nonstatic "rotating-wall" asymmetries can exert a positive torque, causing inward transport and compression, which has enabled the improved confinement of pure-ion [10], pure-electron [11], and pure-positron plasmas [12].

Despite more than 20 years of investigation, the mechanism responsible for asymmetry-induced transport in non-neutral plasmas remains uncertain. Previous measurements [13] have motivated a description of the transport properties in terms of the dimensionless plasma "rigidity," $\mathcal{R} \equiv \overline{f}_b/f_E$, which is the ratio of the axial bounce frequency of a thermal particle to the average azimuthal $\mathbf{E} \times \mathbf{B}$ rotation frequency of the plasma. For electrons, the rigidity depends upon the confining magnetic field B, and the plasma length L, density n, and temperature T as

$$\mathcal{R} \approx 14.6\left[\frac{B}{1\ \mathrm{kG}}\right]^1\left[\frac{L}{10\ \mathrm{cm}}\right]^{-1}\left[\frac{n}{10^7\ \mathrm{cm}^{-3}}\right]^{-1}\left[\frac{T}{1\ \mathrm{eV}}\right]^{1/2}. \tag{1}$$

In this Letter, we present new measurements of cross-magnetic-field transport from applied electrostatic asymmetries. The data span a wide range in plasma rigidity ($1 \le \mathcal{R} \le 140$), allowing us to clearly identify two regimes of asymmetry-induced transport: "$\mathcal{R}^{-2}$" transport and "B-independent" transport. For low rigidity plasmas ($1 \le \mathcal{R} \le 10$), the net expansion rate $\Delta\nu$ due to an applied asymmetry of strength V_a is found to follow the simple formula $\Delta\nu = (7\ \mathrm{s}^{-1})[V_a/1\ \mathrm{V}]\mathcal{R}^{-2}$. As the rigidity is increased into the range $\mathcal{R} \gtrsim 10-20$, this $\mathcal{R}^{-2}$ transport ceases abruptly, and the scalings with plasma parameters and asymmetry strength change. For high rigidity plasmas ($\mathcal{R} > 20$), we observe B-independent transport that does not depend directly on the rigidity and scales with asymmetry strength as $\Delta\nu \propto V_a^2$.

Transport measurements were conducted using two similar Penning-Malmberg traps: "CamV" (shown in Fig. 1) and "EV." Both traps consist of a series of hollow conducting cylinders of radius $r_w = 3.5$ cm (or 3.81 cm), which reside in ultrahigh vacuum $P \approx 10^{-10}$ Torr. Axial confinement is energetically assured by applying voltages ($V_c = -100$ V) to two "confinement cylinders," and radial confinement is provided by a uniform axial magnetic field ($0.1 \le B \le 10$ kG). The length of the plasma ($10 \le L \le 40$ cm) is varied by varying the number of grounded cylinders between the two confinement cylinders. For the present experiments, we trapped $3 \times 10^8 \le N_{\mathrm{tot}} \le 3 \times 10^9$ electrons in a cylindrically symmetric plasma (i.e., $\lambda_D < r_p, L$) of radius $1.5 \le r_p \le 2.0$ cm with an average density of $3 \times 10^6 \le n \le 1.6 \times 10^7$ cm^{-3} and temperature of $0.6 \le T \le 4$ eV. The bounce frequency of a thermal electron $\overline{f}_b \equiv (T/M_e)^{1/2}/2L$ was varied over the range $6 \times 10^5 \le \overline{f}_b \le 3 \times 10^6$ Hz, and the $\mathbf{E} \times \mathbf{B}$ rotation frequency $f_E \equiv cE_r/rB \approx ecn/B$ was varied over the range $8 \times 10^3 \lesssim f_E \lesssim 8 \times 10^5$ Hz. The electron-electron collision frequency was always small compared to the bounce frequency (i.e., $\nu_{ee} < \overline{f}_b/1000$), spanning the range $12 \le \nu_{ee} \le 260$ s^{-1}.

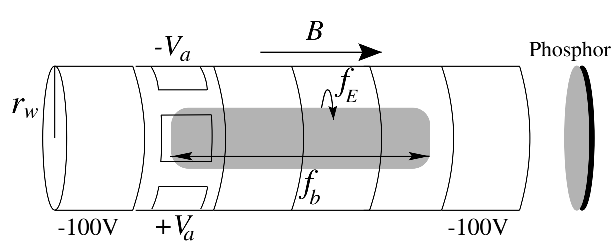

FIG. 1. Schematic of cylindrical trap electrodes showing an applied $m_\theta = 1$ asymmetry of strength V_a.

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We create an azimuthal asymmetry that principally varies as $\phi_a(r,\theta) \propto V_a e^{im_\theta \theta}(r/r_w)^{m_\theta}$, by applying static voltages ($0.05 \leq V_a \leq 40$ V) to four 60° wall sectors of length $L_{\rm sec} = 3.9$ cm (or 4.8 cm), which are located at one end of the plasma. In this paper, we present measurements of transport due to an $m_\theta = 1$ asymmetry, with the voltages on the four sectors being $+V_a/0/-V_a/0$. (Similar transport scalings have also been obtained for $m_\theta = 2$ and $m_\theta = 4$ [14].) In addition to the azimuthal and radial variation, the asymmetry also varies in the axial direction since it is applied only over a portion of the plasma (i.e., $L_{\rm sec}/L < 1$); it may be thought of as consisting of many modes which vary as $e^{i\pi m_z z/L}$, with axial mode number $m_z = 0, 1, 2, \ldots$.

The experiments consist of inject-and-manipulate/perturb-and-hold/dump-and-measure cycles. First, electrons injected from a hot tungsten filament [15] are manipulated to produce density and temperature profiles which are nearly uniform in radius. Then, the asymmetry is ramped to V_a in 1 ms, held at V_a for a time of 10 ms to 2 s, and then ramped back to 0 in 1 ms. Finally, at a time t after injection, the plasma is dumped out one end of the trap onto a phosphor screen (CamV) or a Faraday cup (EV). This destructively measures the z-integrated charge per unit area $Q(r,\theta,t)$, from which we calculate the z- and θ-averaged density as $n(r,t) \equiv \langle Q(r,\theta,t)\rangle_\theta / L$. Here, L is determined by the confinement cylinders and voltages. The high degree of shot-to-shot reproducibility in these systems ($\delta n/n < 0.01$) allows us to measure radial transport from the change in $n(r,t)$ determined by holding different (yet nearly identical) plasmas for different periods of time.

We calculate the radial flux of particles as $\Gamma(r,t) \equiv -\frac{1}{r}\int_0^r dr'\, r' \frac{d}{dt} n(r',t)$, and the mean-square radius as

$$\langle r^2\rangle(t) \equiv \frac{\int_0^{r_w} 2\pi r\, dr\, n(r,t) r^2}{\int_0^{r_w} 2\pi r\, dr\, n(r,t)}. \quad (2)$$

We independently vary the plasma parameters and asymmetry strength and quantify the transport with the "net expansion rate" $\Delta\nu(B,L,n,T;V_a)$ [16] defined as

$$\Delta\nu \equiv \nu - \nu(V_a = 0), \quad (3)$$

with

$$\nu \equiv \frac{1}{\langle r^2\rangle}\frac{d}{dt}\langle r^2\rangle, \quad (4)$$

and where $\nu(V_a = 0)$ is the background rate due to inherent trap asymmetries. The total angular momentum of these electron plasmas is predominantly electromagnetic rather than kinetic and can be approximated as $P_\theta(t) \approx (eB/2c)N_{\rm tot}[r_w^2 - \langle r^2\rangle(t)]$. Here, $N_{\rm tot}$ is constant, so the expansion rate ν is proportional to $(d/dt)P_\theta$, and thus $\Delta\nu$ provides a direct, quantitative measure of the torque due to the applied asymmetry.

In Fig. 2, we show changes in $n(r)$ and $\langle r^2\rangle$ due to $m_\theta = 1$ asymmetries applied to a low rigidity plasma ($\mathcal{R} \approx 5$). Figure 2(a) shows the average density decreasing across the bulk of the plasma as particles move radially outward due to a $V_a = 3$ V asymmetry. Figure 2(b) shows that $\langle r^2\rangle$ increases linearly with time at a rate that is larger for larger applied voltages.

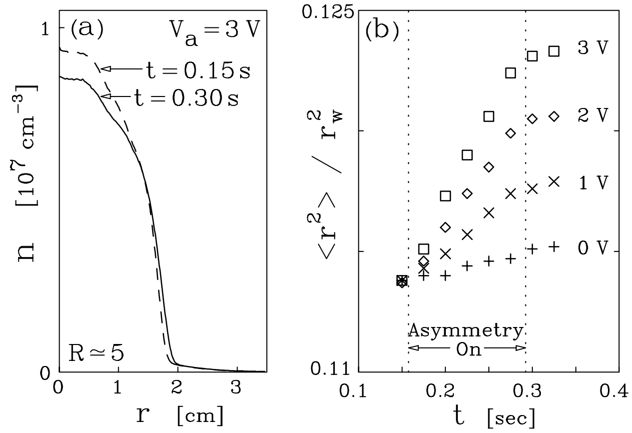

FIG. 2. (a) Change in density profile due to an $m_\theta = 1$ asymmetry of strength $V_a = 3$ V. (b) Increase in $\langle r^2\rangle$ versus time due to applied asymmetries of various strengths.

In Fig. 3, we show the two different asymmetry strength scalings for the two different transport regimes. For high rigidity plasmas (e.g., $\mathcal{R} = 62$), the expansion rate increases with applied voltage as $\Delta\nu \propto V_a^2$. This quadratic scaling holds even for applied voltages at the wall that are larger than the space charge potential of the plasma (here

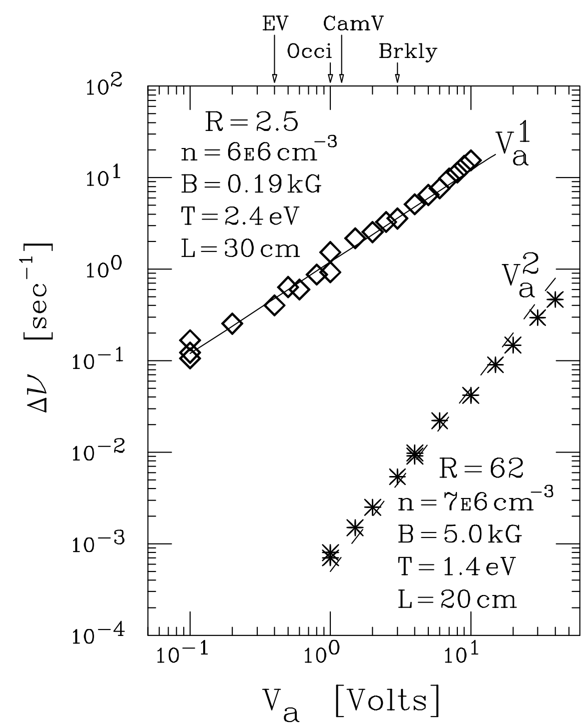

FIG. 3. Net expansion rate $\Delta\nu$ vs asymmetry strength V_a for a low rigidity plasma ($\mathcal{R} = 2.5$) and a high rigidity plasma ($\mathcal{R} = 62$).

$\phi_p \approx 20$ V). For low rigidity plasmas (e.g., $\mathcal{R} = 2.5$), the scaling is different and the expansion rate increases with applied voltage as $\Delta\nu \propto V_a^1$.

This V_a^1 scaling holds over 2 orders of magnitude around the "effective strengths," $V_{\rm trap}$, of inherent asymmetries in typical Penning-Malmberg traps. Here $V_{\rm trap}$ is defined as the applied voltage which doubles the background expansion rate; that is, $\nu(V_a = V_{\rm trap}) = 2\nu(V_a = 0)$. The arrows at the top of Fig. 3 indicate estimates of $V_{\rm trap}$ for the EV and CamV machines [14], as well as published estimates for similar traps at Occidental College [17] and The University of California at Berkeley [18]. In each machine, $V_{\rm trap}$ can vary depending upon how well the magnetic field is aligned with the trap axis, and also upon which cylinders are used for the confinement region. For instance, including the sectored ring in EV (as done here) increases the inherent asymmetry about 7×, from $V_{\rm trap} \approx 0.06$ V to $V_{\rm trap} \approx 0.4$ V [14].

The local particle flux $\Gamma(r)$ measured in low rigidity plasmas also generally depends linearly on the local vacuum field $\phi_a(r)$, i.e., $\Gamma(r) \propto \phi_a^1(r)$ [14,19]. That is, for applied asymmetries with $m_\theta = 1$, 2, or 4, the flux varies with radius as $\Gamma(r) \propto r^{m_\theta}$, matching the radial variation of $\phi_a(r)$. In contrast, measurements on high rigidity plasmas show fluxes varying with higher powers of r; however, this dependence is not demonstrably related to mode number m_θ. Additionally, for weak asymmetries ($V_a \lesssim 1.0$ V), the $\Gamma(r) \propto \phi_a^1(r)$ scaling does not hold near the center of low rigidity plasmas ($r \lesssim 0.2$ cm) [14]. In this limited case, the scaling is closer to $\Gamma(r) \propto \phi_a^2(r)$ [20]. These results suggest that the $\Delta\nu \propto V_a^1$ scaling may be due to a "saturated" response which occurs even for relatively weak fields [i.e., for $|e\phi_a(r_p)| \ll T$].

In Fig. 4, we plot the expansion rate at constant applied asymmetry, $\Delta\nu(V_a = 1\text{ V})$, versus plasma rigidity. For $\mathcal{R} < 10$, the expansion rate decreases with rigidity approximately as $\Delta\nu \propto \mathcal{R}^{-2}$, giving $\Delta\nu \propto B^{-2}L^2n^2T^{-1}$. The increase in transport with plasma length is the most counterintuitive aspect of this scaling, since for longer plasmas the asymmetry is applied over a smaller fraction of the plasma ($L_{\rm sec}/L$).

The $\mathcal{R}^{-2}$ transport mechanism ceases abruptly in the range $10 < \mathcal{R} < 20$, with a 100× drop in $\Delta\nu$ over a 2× increase in $\mathcal{R}$. This "turnoff" is not understood but suggests that if $\mathcal{R} > 20$, the bounce invariant $J_z \equiv \oint dz\, v_z$ is broken only by electron-electron collisions; whereas if $\mathcal{R} < 20$, J_z is also broken by "collisions" with the asymmetric field.

For high rigidity plasmas ($\mathcal{R} > 20$), Fig. 4 shows that the expansion rate does not depend directly on the rigidity. Instead, the transport in this regime scales roughly as $\Delta\nu \propto B^0L^0n^{-1}T^{-1}$ [14]. For this scaling, the most striking aspect is that the cross-magnetic-field transport does not decrease significantly with increasing B. Interestingly, in other experiments this B-independent transport has been found to have the same strength when the asymmetry is applied over the entire containment length (i.e., $L_{\rm sec}/L \geq 1$, giving $m_z = 0$), whereas the $V^1\mathcal{R}^{-2}$ transport occurs only when the asymmetry is applied over a portion of the plasma (i.e., $L_{\rm sec}/L < 1$ or $m_z \neq 0$) [14].

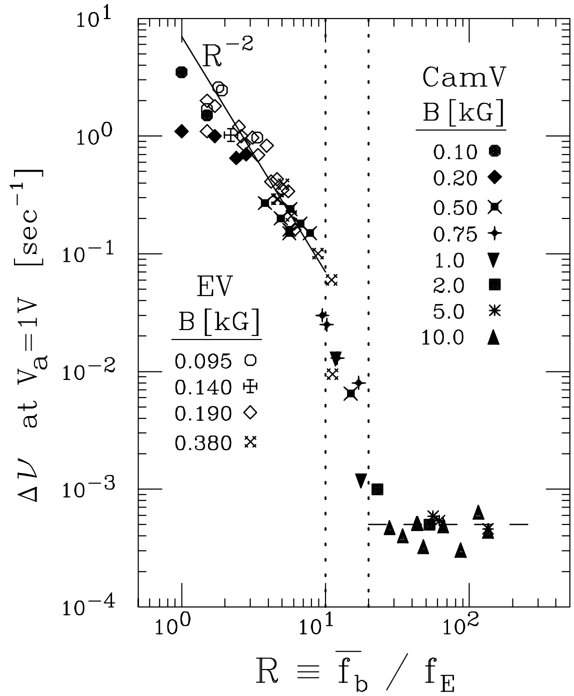

FIG. 4. Net expansion rate $\Delta\nu$ at $V_a = 1$ V vs the plasma rigidity $\mathcal{R}$, measured using two different traps.

The identification of two different transport regimes provides an empirical framework into which other measurements also fit. Previous experiments at The University of California at San Diego showed transport due to inherent asymmetries scaling as $\tau_m^{-1} \propto B^{-2}L^2n_{\rm o}^2T^{-1} \propto \mathcal{R}^{-2}$ for $0.1 \leq \mathcal{R} \leq 10$ [13,14,21]. Here, τ_m is the time for the central plasma density $n_{\rm o}(t)$ to decrease to 1/2 its original value, i.e., $n_{\rm o}(\tau_m) \equiv n_{\rm o}(0)/2$. In all, this $\mathcal{R}^{-2}$ transport has been observed in five different machines over a wide range in magnetic field ($0.03 \leq B \leq$ 60 kG), plasma length ($4 \leq L \leq 80$ cm), temperature ($0.1 \leq T \leq 10$ eV), and central density ($10^6 \leq n_{\rm o} \leq 10^{10}$ cm^{-3}). In these previous experiments, deviations from the $\mathcal{R}^{-2}$ scaling were found to occur in two separate regimes: highly rigid plasmas [14] with $\mathcal{R} \gtrsim 10$ (which is a result consistent with Fig. 4) and highly collisional electron or ion plasmas with ν_{ee} or $\nu_{ii} \gtrsim \overline{f}_b/100$ [11,21] (which is a regime not studied in the current experiments).

Using a trap with a biased central wire, inherent asymmetry transport of low density ($n \approx 10^5$ cm^{-3}) annular electron clouds has also been found to scale as $B^{-2}L^2$ [17]. Because of the similarity in scaling, we believe that this transport mechanism is probably the same as the $\mathcal{R}^{-2}$ mechanism. These low density results [as well as the $\Gamma(r) \propto r^{m_\theta}$ results discussed above] suggest that shielding and other collective plasma effects are not essential to the $\mathcal{R}^{-2}$ transport process.

Motivated by the $\mathcal{R}^{-2}$ scaling and by theories of ion transport in (neutral plasma) tandem mirror machines [22,23], transport theories for low rigidity plasmas have often invoked bounce-resonant particles (i.e., $m_z f_b = m_\theta f_E$). Indeed, there has been some recent experimental evidence of bounce-resonant phenomena in pure-electron plasmas [24,25]. However, we find that bounce-resonant transport theory [26] does not agree with the measurements presented here. In particular, the theory does not predict the observed V_a^1 scaling, the abrupt decrease in transport at $10 < \mathcal{R} < 20$, or the observed B-independent regime.

Despite a clear empirical scaling, an adequate theoretical description of $\mathcal{R}^{-2}$ transport in low rigidity plasmas has not yet been found. The reason may be that most transport theories assume a "small" perturbation, whereas the asymmetries applied here (as well as inherent asymmetries) may trap or completely reflect some low energy particles.

For moderate to high rigidity plasmas ($\mathcal{R} \gtrsim 10$), other applied asymmetry experiments found transport scalings of $\tau_m^{-1} \propto B^{-0.65} n_o^{-0.7} T^0 V_a^2$ [18]. Presumably, the transport mechanism is the same as the so-called B-independent mechanism presented here. Differences between the respective scalings (e.g., $B^{-0.65}$ compared to B^0) are most likely due to the fact that a single scaling was obtained in Ref. [18] for a data set that included the transition region of $10 < \mathcal{R} < 20$.

We believe that the B-independent transport is similar to "rotational-pumping" transport [27,28], which is due to compressional viscosity acting on length changes arising from plasma rotation in asymmetric confining fields. Rotational-pumping theory quantitatively matches experiments which measure the transport in off-center plasmas [28], but the model of asymmetries causing only simple length changes [27] may be inadequate to describe the transport from the asymmetries applied here.

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[1] J. H. Malmberg and J. S. deGrassie, Phys. Rev. Lett. **35**, 577 (1975).

[2] For recent results, see *Non-Neutral Plasma Physics III,* edited by J. J. Bollinger, R. L. Spencer, and R. C. Davidson, AIP Conf. Proc. No. 331 (AIP, New York, 1999).

[3] E. M. Hollmann, F. Anderegg, and C. F. Driscoll, Phys. Rev. Lett. **82**, 4839 (1999); F. Anderegg *et al.,* Phys. Rev. Lett. **78**, 2128 (1997); J. M. Kriesel and C. F. Driscoll, in Ref. [2], p. 266.

[4] D. Durkin and J. Fajans, Phys. Fluids **12**, 289 (2000); K. S. Fine *et al.,* Phys. Rev. Lett. **75**, 3277 (1995).

[5] T. B. Mitchell *et al.,* Science **282**, 1290 (1998).

[6] E. H. Chao *et al.,* in Ref. [2], p. 278.

[7] A. J. Peurrung, R. T. Kouzes, and S. E. Barlow, Mass Spectrom. Ion Processes **157/158**, 39 (1996).

[8] W. M. Itano and N. F. Ramsey, Sci. Am. **269**, No. 1, 56 (1993); J. N. Tan *et al.,* IEEE Trans. Instrum. Meas. **44**, 144 (1995).

[9] G. Gabrielse *et al.,* Phys. Lett. B **455**, 311 (1999); R. G. Greaves and C. M. Surko, Phys. Plasmas **4**, 1528 (1997).

[10] X.-P. Huang *et al.,* Phys. Rev. Lett. **78**, 875 (1997).

[11] E. M. Hollmann, F. Anderegg, and C. F. Driscoll, Phys. Plasmas **7**, 2776 (2000).

[12] R. G. Greaves and C. M. Surko (private communication).

[13] C. F. Driscoll and J. H. Malmberg, Phys. Rev. Lett. **50**, 167 (1983); C. F. Driscoll, K. S. Fine, and J. H. Malmberg, Phys. Fluids **29**, 2015 (1986); A. C. Cass *et al.,* Bull. Am. Phys. Soc. **39**, 1737 (1994).

[14] J. M. Kriesel, Ph.D. thesis, University of California, San Diego, 1999.

[15] J. M. Kriesel and C. F. Driscoll, Phys. Plasmas **5**, 1265 (1998).

[16] The expansion rate may also depend upon other parameters which we did not vary, such as r_p and L_{sec}.

[17] D. L. Eggleston, Phys. Plasmas **4**, 1196 (1997).

[18] J. Notte and J. Fajans, Phys. Plasmas **1**, 1123 (1994).

[19] J. Kriesel and C. F. Driscoll, in Ref. [2], p. 256.

[20] It is still possible that $\Delta\nu \propto V_a^1$ in this case since $\Delta\nu$ is insensitive to transport near $r = 0$.

[21] B. P. Cluggish, Ph.D. thesis, University of California, San Diego, 1995.

[22] D. Ryutov and G. Stupakov, JETP Lett. **26**, 174 (1978).

[23] R. Cohen, Nucl. Fusion **19**, 1579 (1979).

[24] E. Gilson and J. Fajans, in Ref. [2], p. 250; D. L. Eggleston, *ibid.,* p. 241.

[25] B. P. Cluggish, J. R. Danielson, and C. F. Driscoll, Phys. Rev. Lett. **81**, 353 (1998).

[26] D. L. Eggleston and T. M. O'Neil, Phys. Plasmas **6**, 2699 (1999).

[27] S. Crooks and T. M. O'Neil, Phys. Plasmas **2**, 355 (1995).

[28] B. P. Cluggish and C. F. Driscoll, Phys. Rev. Lett. **74**, 4213 (1995).