Geometric Electro-Magnetic Field Equations for Particles with Charge and Dipole Spin

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The geometric and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) Force and Torque interactions between moving Particles with bound Charge and Dipole Spin .

Math Point / Real Particle

The 3D Euclidean geometry is properly describied by the **Grassmann extension algebra** with 4 Basis Elements **{ 3 Vectors, 1 Point }**, here denoted **G3p1** .
This generates structures for bound Particles, in addition to lines, planes, and volumes.

This broader geometric algebra can incorporate the two fundamental lengths in Physics :
(1) the "classical radius of the electron", denoted $R_e \equiv e^2/m_e c^2 = 2.82$ pm (10^{-15} m), which scales the electric interaction energy between *two* charged particles; and
(2) the "Compton wavelength", here denoted $D_v \equiv \bar{\lambda}_c \equiv \hbar c / m_e c^2 = 386.$ pm, which scales the magnetic Dipole Spin of a *single* electron .

With this full algebra, **Forces** betwen 2 or more particles can add to become a **Torque**, and two or more linear **Flows** can add to become a **Circulation** , giving a basis for **Dipole Spin**. This can *not* be expressed in the "2D tangent space" of Clifford Algebra Cℓ3 or Vector Algebra.

With similar clarity, the G3p1 algebra describes the motional "transformations" between vector electric E and bi-vector magnetic B , through 1st order (v / c) motion of an Origin Point, *without* 2nd order space-time algebra effects.

Duality : It Takes Two to Tangle

The triumph of Maxwell's Equations is the description of E-M waves, propagating outward from "charged particles" at fundamental speed **c** . These are generally viewed as either "light waves", or as "*Photons* with spin", expressing the "wave / particle" duality of modern physics.

In G3p1, the isolated spherical E-M wave ("PhoTinoH") emanating outward from a *single* Particle has invariant energy density $\vec{E}^2 + \hat{B}^2 = 0$, because the bi-vector magnetic field squares to negative. That is, 1-sided emanations are "potential", or "nascent", becoming energetically "bipotent" only when coupled to an oppositely propagating PhoTinoH from a second Particle.

That is, , the *superposition of two counter-propagating PhoTinoH* is readily seen to give the inter-particle Force and Torque transfers, incorrectly attributed to a single preternatural *Photon* . Moreover, the symmetry of the two PhoTinoH enables the "equal, opposite, and simultaneous" "Newtonian ideal interaction", despite the finite propagation delays from speed **c** .

In the 3D algebra of G3p1, the Particle-Field-Particle equations display "CP$\mathcal{H}$" (rather than CPT) symmetries: 3 geometric Parity "flips" $\check{u}_x \check{u}_y \check{u}_z$; plus particle Charge $\check{u}_Q$; and particle $\mathcal{H}$elicity $\check{u}_{\mathcal{H}}$. These sufficiently distinguish 4 particles { e-,p+, e+,p- } , and show the scalar field φ(x,t) to be dual to the scalar field ψ(x,t) describing tri-vector "Twist" .

Moreover, geometric $\mathcal{H}$elicity shows that the velocity $\vec{\beta}$ of an electron is always perpendicular to the symmetry plane of the Dipole Spin ; and necessarily, $|\beta| > 0$. Here, the dynamics of closely interacting "spins" may shed light on the "quantum of uncertainty", expressed as $\Delta x \gtrsim D_v / \beta_x$.

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Geometric Linear Algebras

Geometric Product : $A \,\mathring{\wedge}\, B = A \circ B + A \wedge B$ (contract //, extend ⊥)

G3p1 ≡ Grassmann Extension Algebra
3+1 Basis Elements : **3 Vectors +1 Point** or: 3+1 Points

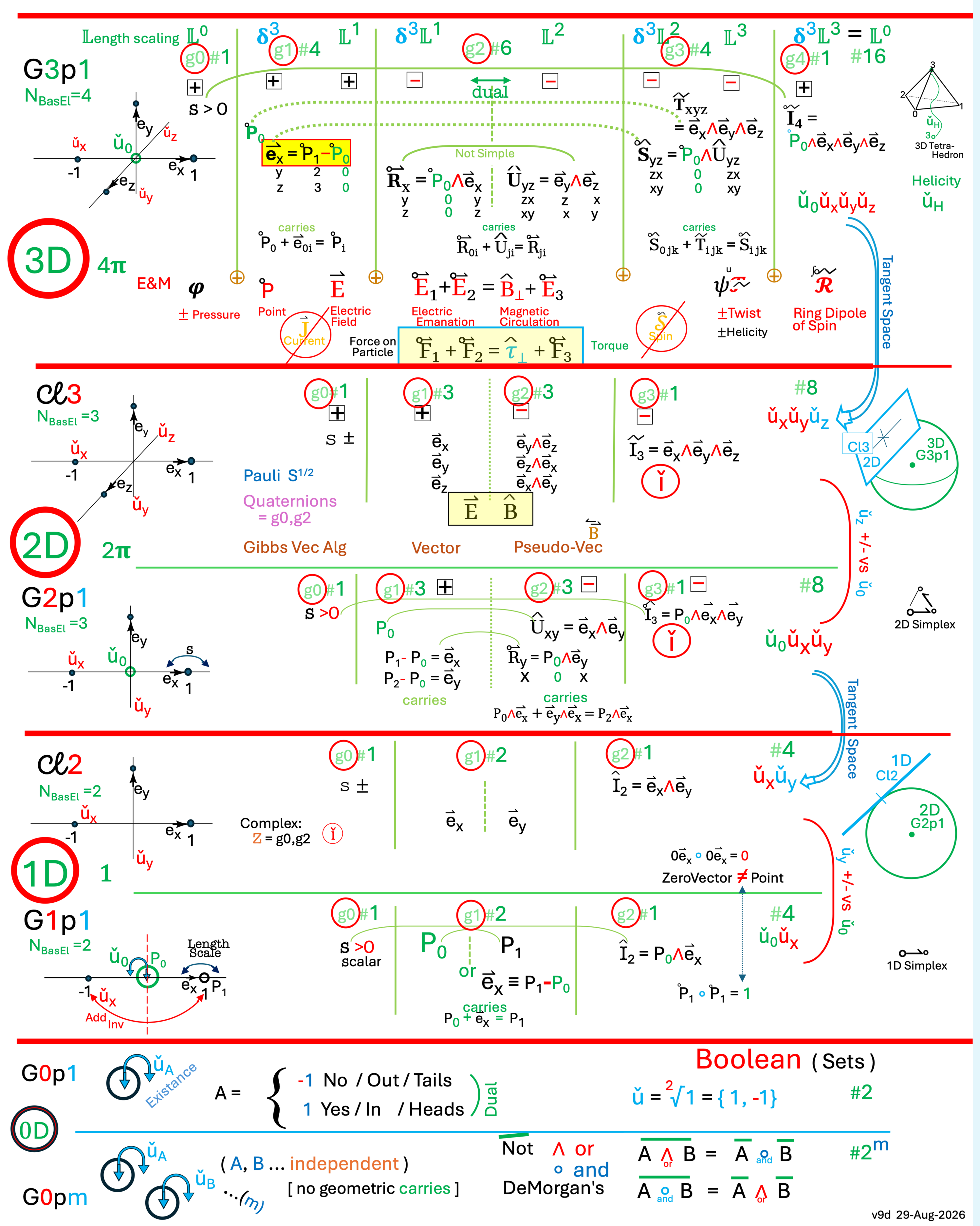
3-Dimensional Affine Space (points $\mathcal{A}$, carries $\mathcal{V}^{(3)}$)

G3p1 gives the full Division Algebra *needed* for geometric Particle/Field interactions in real 3D-Space

Cℓ3 ≡ Clifford Geometric Algebra (Hestenes)
3 Basis Elements: 3 **Vectors**

Wedge, [Cross], Extension, Protrusion: ∧ is ⊥
Dot, [Scalar], Contraction, Adherence: ∘ is //

$\check{u}^2 = 1$ → ± flip
$\check{I}^4 = 1$ → 90°rot



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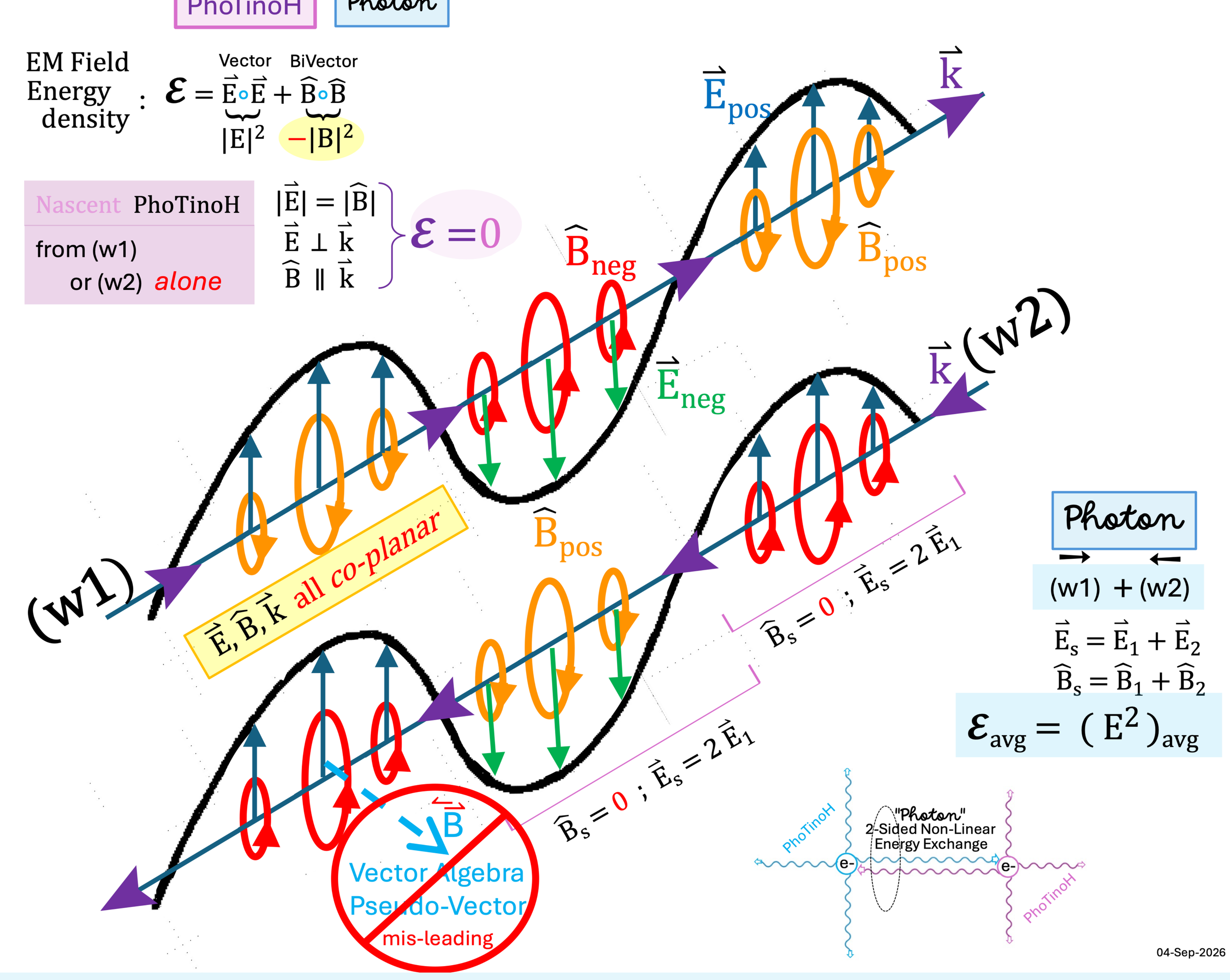
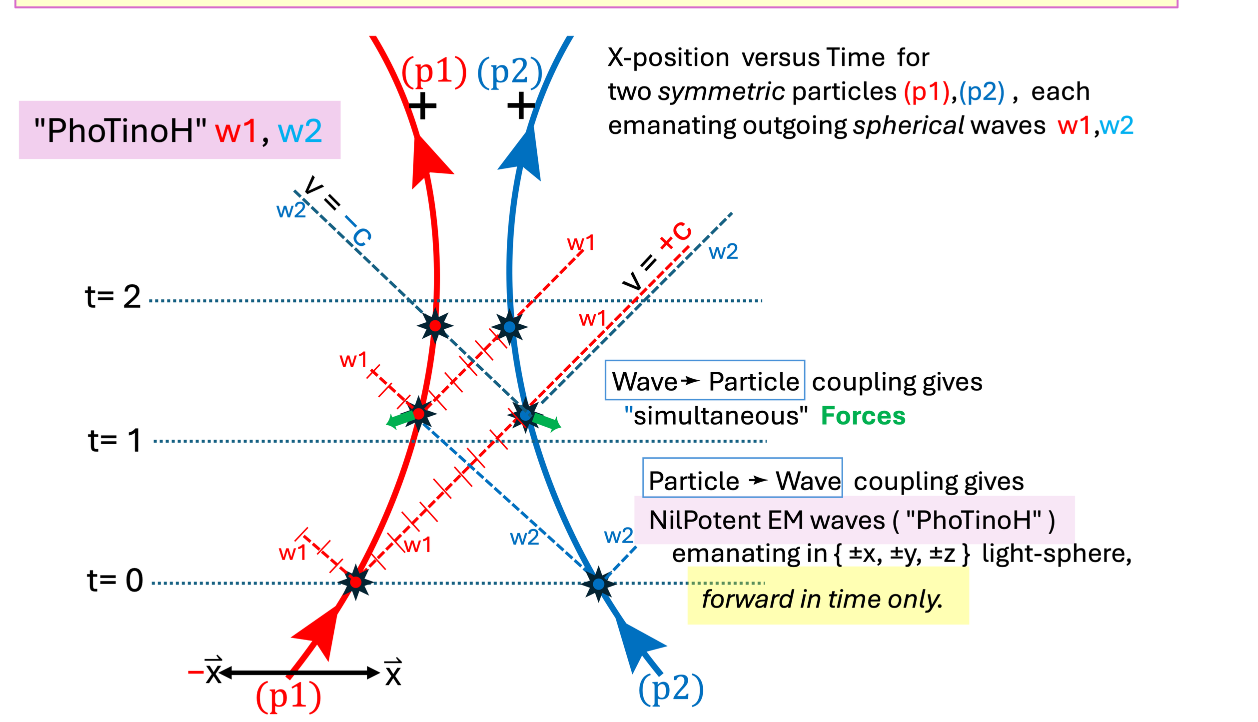
It Takes Two to Tangle



Each Particle p1,p2 continuously *emanates* outgoing EM Fields w1,w2 ; these (*linear*) E, B Fields carry no (non-linear) Energy, eventually being partially *absorbed* by the other Particle, giving *simultaneous, equal, and opposite* Forces between Particles.



Maxwell / Lorentz Equations with Charge and Dipole Spin

Traditional 2D Fluid Algebras, no Time Delay

Cℓ3 Geometric Algebra (Clifford / Hestenes, 2D Tangent Space) ⇔ Std. Vector Algebra (Gibbs / Heavyside, 1D+)

	∘ Dot, ∧ Wedge		∘ Dot, × Cross
g0	$\nabla \circ \vec{E} =$	⇔	$\nabla \circ \vec{E} = 4\pi(\rho_+ - \rho_-)$ (2-fluids)
g1	$-\nabla \circ \hat{B} =$	⇔	$\nabla \times \vec{B} = \frac{\partial}{\partial ct}\vec{E} + 4\pi \vec{J}_c$
g2	$\nabla \wedge \vec{E} =$	⇔	$\nabla \times \vec{E} = -\frac{\partial}{\partial ct}\vec{B}$
g3	$\nabla \wedge \hat{B} =$ (bi-vec)	⇔	$\nabla \circ \vec{B} = 0$ (pseudo-vector)

Q; $\mathcal{T}$ Twist, $\mathcal{H}$elicity

Geometric Product

$A \,\mathring{\wedge}\, B \equiv A \wedge B + A \circ B$
∧ Orthogonal Extension
∘ Parallel Contraction

Cl3 and G3p1 build graded structures with the Geometric Product.
G3p1 builds up to Vorticity.

Units

$\mathbb{L}$ [meter]
$\mathbb{T} \equiv \mathbb{L}/c$ *299.79 x10^6 [sec]
$\mathcal{E}_0 \equiv m_e c^2$ ~ 511. [keV]

Length scales of E & M

$R_e = \frac{e^2}{\mathcal{E}_0}$ ~ 2.82 x10^{-15} m

$D_v = \frac{\hbar c}{\mathcal{E}_0}$ ~ 386. x10^{-15} m

Standard "Gibbs Vector Algebra" becomes *inconsistent* at pseudo-vector, and can not represent 2 fluids .
Cl3 has 3 Basis Vectors, representing a consistent 2D Tangent Space, but *cannot* represent 3D Particles or Spin.

CP$\mathcal{H}$ Symmetry

$\check{u} = \{+,-\}$

Geometric Parity $\check{u}_0\ \check{u}_x\ \check{u}_y\ \check{u}_z$

Bound Particles	e-	p+	e+	p-
Charge $\check{u}_Q$	−	+	+	−
$\mathcal{H}$elicity $\check{u}_H$	+	+	−	−

G3p1 linear algebra connects 3-D Fields to Particle Sources with Charge and 3-D Dipole Spin.

1-Sided Linear Algebra, giving "Nascent" Fields from Source Particles P_s .

$\mathrm{P}_s(\vec{x}_s, \tau_s)$

$\vec{r}_{sf} = \mathrm{P}_s - \mathrm{P}_f$; R = $|r_{sf}|$

$\vec{\beta} = \frac{\partial}{\partial ct}(\mathrm{P}_s - \mathrm{P}_0)$

$\vec{\beta}_{/\!/} = (\vec{\beta} \circ \check{r})\,\check{r}$

$\vec{\beta}_{\perp} = (\vec{\beta} \wedge \check{r}) \circ \check{r}$

$\kappa_{/\!/} = 1 - \vec{\beta}_{/\!/} \circ \check{r}$

g0 $\left[\frac{\#e}{R}\mathbb{1}\right]$ — Nascent Field, ±Pressure; Emanation; Fluidize; Source Particles @(x_s, τ_s); Monopole Charge; Dipole Charge (if); Distant Time Delay

$$\varphi(x_0,t)\,\mathbb{1} \Leftarrow \int d\tilde{x} \,\mathring{\wedge} \sum_{s}^{s\neq f}\left\{\frac{e\,\check{u}_s \mathrm{P}_s}{\kappa_{/\!/} R_{sf}} \,\mathring{\wedge}\, \mathbb{1}\left(1 + \frac{\vec{d}_s \circ \vec{r}_{sf}}{R_{sf}}\right)\right\}_{@\tau_s = t - R_{sf}/c}$$

g1p $\left[\frac{1}{\mathbb{LL}}\frac{\#e}{R}\right]$
$$\frac{\nabla^2}{4\pi}\varphi(x_s,t)\,\mathbb{1} = \frac{\vec{\nabla}}{4\pi}\circ \vec{E}(\sim x_s) \Leftarrow e\,\check{u}_s \langle \mathfrak{R}_s \rangle_{g1p}$$

Toroidal Ring of Charge

g1 $\left[\frac{\#e}{R\mathbb{L}}\right]$
$$\vec{E}(x) = -\vec{\nabla}\wedge\varphi(x)$$

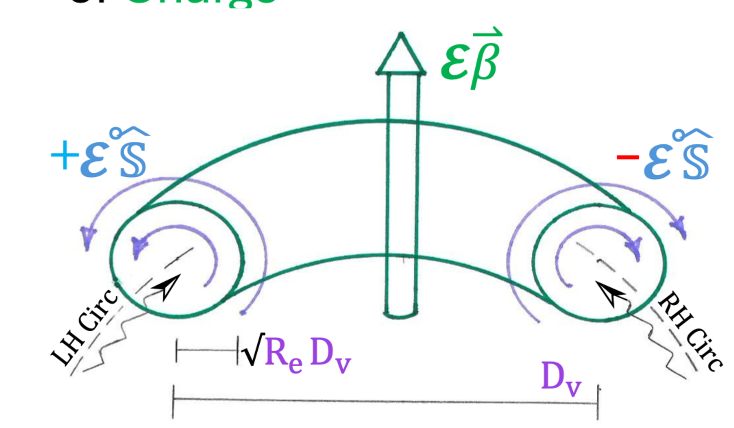
Vacuum All (x,t)

g1 $\left[\frac{1}{ct}\frac{\#e}{R\mathbb{L}}\right]$
$$\frac{\partial}{\partial ct}\vec{E}(x) = \frac{\vec{\nabla}}{4\pi}\circ\hat{B}(x)$$

g2 $\left[\frac{1}{ct}\frac{\#e}{R}\beta\right]$
$$\frac{\partial}{\partial ct}\hat{B}(x) = \vec{\nabla}\wedge\vec{E}(x)$$

g2 $\left[\frac{\#e}{R}\beta\right]$
$$\hat{B}(x) = -\vec{\nabla}\circ\psi(x)\,\widetilde{\mathcal{T}}$$

Toroidal Ring of Dipole Spin



Electron Geometry

g4p $\left[\frac{1}{\mathbb{LL}}\frac{\#e}{R}\beta\right]$
$$\frac{\nabla^2}{4\pi}\psi(x_s,t)\,\widetilde{\mathcal{T}} = \vec{\nabla}\wedge\hat{B}(\sim x_s) \Leftarrow e\,\check{u}_s \langle \check{u}_H \mathfrak{R}_s \rangle_{g4p}$$

g3 $\left[\frac{\#e}{R}\beta\right]$ — Nascent Field, ±Twist; Emanation; Fluidize; Source Particles @(x_s, τ_s); No Monopole Spin; Dipole of Spin; Distant Time Delay

$$\psi(x_0,t)\,\widetilde{\mathcal{T}} \Leftarrow \int d\tilde{x} \,\mathring{\wedge} \sum_{s}^{s\neq f}\left\{\frac{e\,\check{u}_s \mathrm{P}_s}{\kappa_{/\!/} R_{sf}} \,\mathring{\wedge}\, \hat{\mathcal{S}} \,\mathring{\wedge}\left[0 + \frac{D_v}{R_{sf}}\check{u}_{Hs}\beta_{s\perp}\right]\right\}_{@\tau_s = t - R_{sf}/c}$$

2-Sided Energetics

Energy is transferred between Two Particles, by the symmetric Field / Particle interactions , "dual" between Charge & Helicity .

Positional $\left[\pm\mathcal{E}_0 \frac{R_e}{R_{12}} \mathrm{P}\right]$
$$\mathcal{E}_{R12}\,\mathrm{P}(x) = \varphi(x)\,\mathbb{1} \circ e\,u_Q \langle \mathfrak{R}(x)\rangle_{g1p} \quad \left[\frac{\#e}{R} \circ (\pm e)\,\delta^3\right]$$

$$\vec{\mathcal{F}}(x) = -\vec{\nabla}\,\mathcal{E}_{R12}\,\mathrm{P}(x)$$

Relative Orientations $\left[\pm\mathcal{E}_0 \frac{R_e}{R_{12}} \mathrm{P} \frac{D_v}{R_{12}}\right]$
$$\mathcal{E}_{R\theta\theta}\,\mathrm{P}(x) = \psi(x)\,\mathcal{T} \circ e\,u_Q \langle u_H \mathfrak{R}(x)\rangle_{g4p} \quad \left[\frac{\#e}{R}\beta_{\perp}\circ\beta_{\perp}^{1}\,(\pm e)\,\delta^3\right]$$

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