Geometric Electro-Magnetic Field Equations for Particles with Charge and Dipole Spin

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The geometric and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) Force and Torque interactions between moving Particles with bound Charge and Dipole Spin .

Math Point / Real Particle

The 3D Euclidean geometry is properly descriibed by the **Grassmann extension algebra** with 4 Basis Elements **{ 3 Vectors, 1 Point },** here denoted **G3p1 .**
This generates structures for bound Particles, in addition to lines, planes, and volumes.

This broader geometric algebra can incorporate the two fundamental lengths in Physics :
(1) the "classical radius of the electron", denoted $\mathbf{R_e} \equiv e^2/m_e c^2$ = 2.82 pm (10^{-15} m),
which scales the electric interaction energy between *two* charged particles; and
(2) the "Compton wavelength", here denoted $\mathbf{D_v} \equiv \lambda\!\!\!^{-}_c \equiv \hbar c / m_e c^2$ = 386. pm,
which scales the magnetic Dipole Spin of a *single* electron .

With this full algebra, **Forces** betwen 2 or more particles can add to become a **Torque**, and two or more linear **Flows** can add to become a **Circulation** , giving a basis for **Dipole Spin**.
This can *not* be expressed in the "2D tangent space" of Clifford Algebra $C\ell 3$ or Vector Algebra.

With similar clarity, the G3p1 algebra describes the motional "transformations" between vector electric $\mathbf{E}$ and bi-vector magnetic $\mathbf{B}$ **,** through 1st order (v / c) motion of an Origin Point, *without* 2nd order space-time algebra effects.

Duality : It Takes Two to Tangle

The triumph of Maxwell's Equations is the description of E-M waves, propagating outward from "charged particles" at fundamental speed **c** . These are generally viewed as either "light waves", or as "*Photons* with spin", expressing the "wave / particle" duality of modern physics.

In G3p1, the isolated spherical E-M wave ("PhoTinoH") emanating outward from a *single* Particle has invariant energy densiy $\vec{\mathrm{E}}^2 + \widehat{\mathrm{B}}^2 = 0$, because the bi-vector magnetic field squares to negative.
That is, 1-sided emanations are "potential", or "nascent", becoming energetically "bipotent" only when coupled to an oppositely propagating PhoTinoH from a second Particle.

That is, , the *superposition of two counter-propagating PhoTinoH* is readily seen to give the inter-particle Force and Torque transfers, incorrectly attributed to a single preternatural *Photon* .
Moreover, the symmetry of the two PhoTinoH enables the "equal, opposite, and simultaneous" "Newtonian ideal interaction", despite the finite propagation delays from speed **c** .

In the 3D algebra of G3p1, the Particle-Field-Particle equations display "CP$\mathcal{H}$" (rather than CPT) symmetries: 3 geometric Parity "flips" $\check{u}_x \check{u}_y \check{u}_z$; plus particle Charge $\check{u}_Q$; and particle $\mathcal{H}$elicity $\check{u}_{\mathcal{H}}$.
These sufficiently distinguish 4 particles { e-,p+, e+,p- } , and show the scalar field $\boldsymbol{\varphi}(\mathrm{x,t})$ to be dual to the scalar field $\boldsymbol{\psi}(\mathrm{x,t})$ describing tri-vector "Twist" .

Moreover, geometric $\mathcal{H}$elicity shows that the velocity $\vec{\beta}$ of an electron is always perpendicular to the symmetry plane of the Dipole Spin ; and necessarily, $|\beta| > 0$. Here, the dynamics of closely interacting "spins" may shed light on the "quantum of uncertainty", expressed as $\Delta x \gtrsim D_v / \beta_x$.

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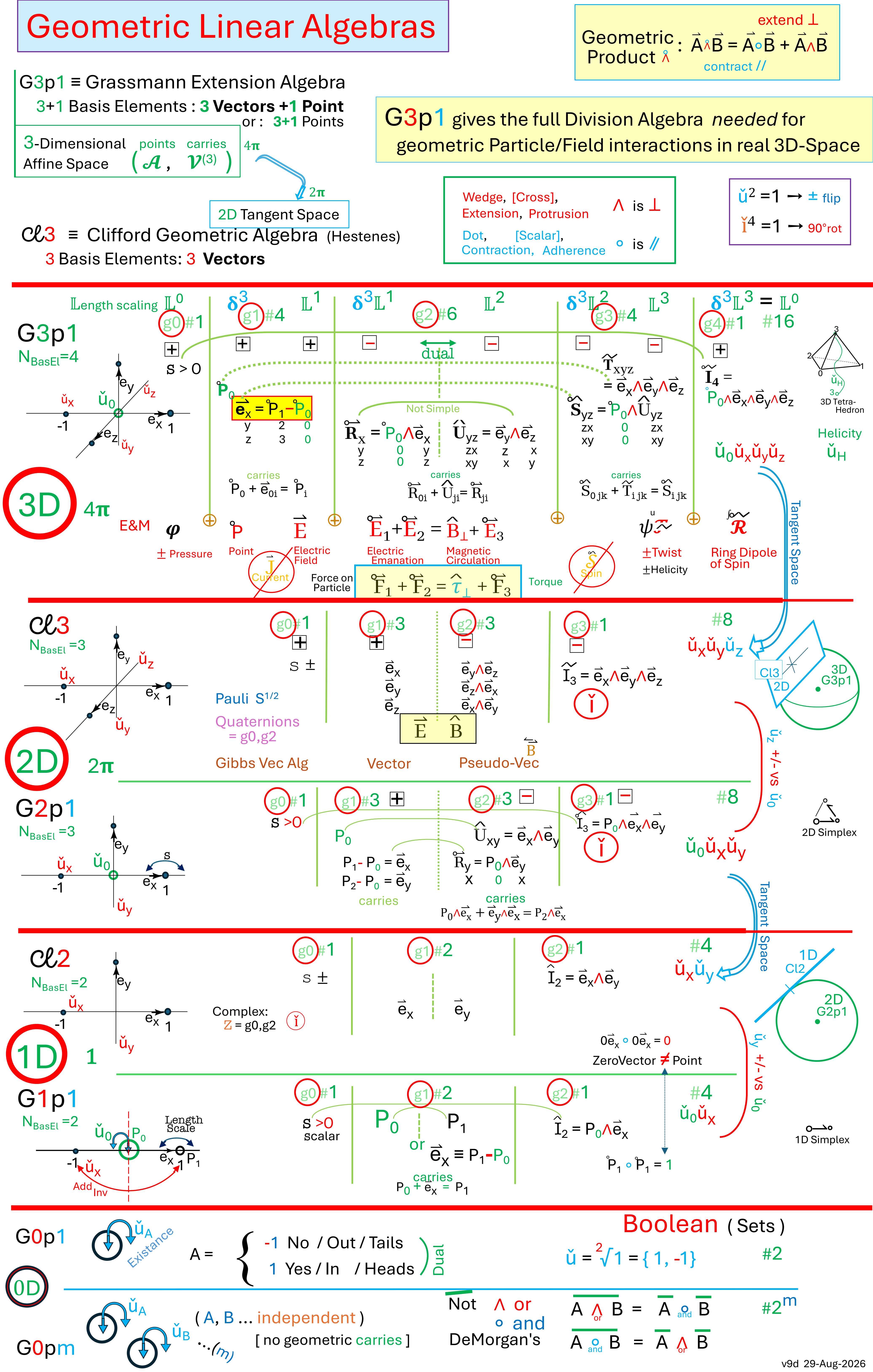
Geometric Linear Algebras



Maxwell / Lorentz Equations with Charge and Dipole Spin

Traditional 2D Fluid Algebras, no Time Delay

Cl3 Geometric Algebra — Clifford / Hestenes, 2D Tangent Space ⇔ **Std. Vector Algebra** — Gibbs / Heavyside, 1D+

	Cl3: ∘ Dot, ∧ Wedge		Std.: ∘ Dot, × Cross
g0	$\nabla\circ\vec{E} =$	⇔	$\nabla\circ\vec{E} = 4\pi\,(\rho_+ - \rho_-)$ (2-fluids)
g1	$-\nabla\circ\hat{B} =$	⇔	$\nabla\times\vec{B} = \frac{\partial}{\partial ct}\vec{E} + 4\pi\,\cancel{\vec{J}/c}$
g2	$\nabla\wedge\vec{E} =$	⇔	$\nabla\times\vec{E} = -\frac{\partial}{\partial ct}\vec{B}$
g3	$\nabla\wedge\hat{B} =$	⇔	$\nabla\circ\vec{B} = 0$
	bi-vec		pseudo-vector

Q

$\mathcal{T}$ — $\mathcal{T}$wist, $\mathcal{H}$elicity

Standard "Gibbs Vector Algebra" becomes *inconsistent* at pseudo-vector, and can not represent 2 fluids .
Cl3 has 3 Basis Vectors, representing a consistent 2D Tangent Space, but *cannot* represent 3D Particles or Spin.

Geometric Product

$A \,\mathring{\wedge}\, B \;\equiv\; A\wedge B \;+\; A\circ B$

∧ Orthogonal Extension
∘ Parallel Contraction

Cl3 and G3p1 build graded structures with the Geometric Product.
G3p1 builds up to Vorticity.

Units

$\mathbb{L}$		[meter]
$\mathbb{T} \equiv \mathbb{L}/c$	$*299.79\times10^{6}$	[sec]
$\mathcal{E}_0 \equiv m_e c^2$	~ 511.	[keV]

Length scales of E & M

$R_e = \frac{e^2}{\mathcal{E}_0} \sim 2.82\times10^{-15}$ m

$D_v = \frac{\hbar c}{\mathcal{E}_0} \sim 386.\times10^{-15}$ m

CP$\mathcal{H}$ Symmetry

$\check{u} = \{+, -\}$

Geometric Parity: $\check{u}_0\ \check{u}_x\ \check{u}_y\ \check{u}_z$

		Bound Particles			
		e−	p+	e+	p−
Charge	$\check{u}_Q$	−	+	+	−
$\mathcal{H}$elicity	$\check{u}_H$	+	+	−	−

G3p1 linear algebra connects 3-D Fields to Particle Sources with Charge and 3-D Dipole Spin.

1-Sided Linear Algebra, giving "Nascent" Fields from Source Particles P_s .

$\mathring{P}_s(\vec{x}_s, \tau_s)$

$\boxed{\vec{r}_{sf} = \mathring{P}_s - \mathring{P}_f}$

$R = |r_{sf}|$

$\boxed{\vec{\beta}} = \frac{\partial}{\partial ct}(\mathring{P}_s - \mathring{P}_0)$

$\vec{\beta}_{/\!/} = (\vec{\beta}\circ\overset{u}{\vec{r}})\,\overset{u}{\vec{r}}$

$\vec{\beta}_{\perp} = (\vec{\beta}\wedge\overset{u}{\vec{r}})\circ\overset{u}{\vec{r}}$

$\kappa_{/\!/} = 1 - \vec{\beta}_{/\!/}\circ\overset{u}{\vec{r}}$

g0 $\left(\frac{\#e}{R}\mathbb{1}\right)$ — Nascent Field, Emanation, Fluidize, Source Particles @(x_s,τ_s), Monopole Charge, Dipole Charge (if), Distant Time Delay:

$$\boldsymbol{\varphi}(x_0,t)\,\mathbb{1} \Leftarrow \int \mathrm{d}\tilde{\vec{x}}\,\mathring{\wedge} \sum_{s}^{s\neq f}\left\{ \frac{e\,\check{u}_s \mathring{P}_s}{\kappa_{/\!/} R_{sf}}\,\mathring{\wedge}\,\mathbb{1}\left(1 + \frac{\vec{d}_s\circ\vec{r}_{sf}}{R_{sf}}\right)\right\}_{@\tau_s = t - R_{sf}/c}$$

±Pressure

g1p $\left(\frac{1}{LL}\frac{\#e}{R}\right)$ $\quad \frac{\nabla^2}{4\pi}\boldsymbol{\varphi}(x_s,t)\,\mathbb{1} = \frac{\vec{\nabla}}{4\pi}\circ\vec{E}(\sim x_s) \Leftarrow e\,\check{u}_s\langle\mathcal{R}_s\rangle_{g1p}$ — Toroidal Ring of Charge

g1 $\left(\frac{\#e}{RL}\right)$ $\quad \vec{E}(x) = -\vec{\nabla}\wedge\boldsymbol{\varphi}(x)$

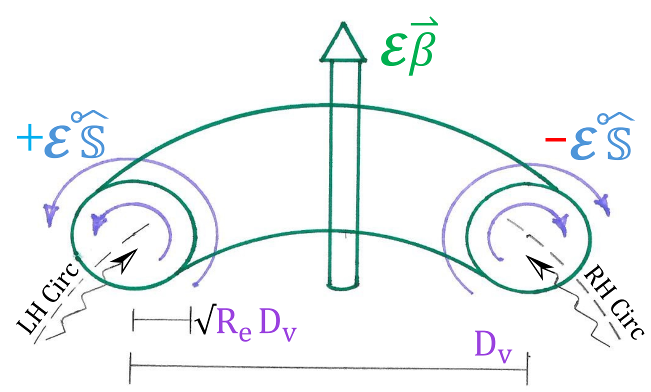
g1 $\left(\frac{1}{ct}\frac{\#e}{RL}\right)$ $\quad \frac{\partial}{\partial ct}\vec{E}(x) = \frac{\vec{\nabla}}{4\pi}\circ\hat{B}(x)$

Vacuum All (x,t)

g2 $\left(\frac{1}{ct}\frac{\#e}{R}\beta\right)$ $\quad \frac{\partial}{\partial ct}\hat{B}(x) = \vec{\nabla}\wedge\vec{E}(x)$

g2 $\left(\frac{\#e}{R}\beta\right)$ $\quad \hat{B}(x) = -\vec{\nabla}\circ\boldsymbol{\psi}(x)\,\tilde{\mathcal{T}}$

g4p $\left(\frac{1}{LL}\frac{\#e}{R}\beta\right)$ $\quad \frac{\nabla^2}{4\pi}\boldsymbol{\psi}(x_s,t)\,\tilde{\mathcal{T}} = \vec{\nabla}\wedge\hat{B}(\sim x_s) \Leftarrow e\,\check{u}_s\langle\check{u}_H\mathcal{R}_s\rangle_{g4p}$ — Toroidal Ring of Dipole Spin



Electron Geometry

g3 $\left(\frac{\#e}{R}\beta\right)$ — Nascent Field, Emanation, Fluidize, Source Particles @(x_s,τ_s), No Monopole Spin, Dipole of Spin, Distant Time Delay:

$$\boldsymbol{\psi}(x_0,t)\,\tilde{\mathcal{T}} \Leftarrow \int \mathrm{d}\tilde{\vec{x}}\,\mathring{\wedge} \sum_{s}^{s\neq f}\left\{ \frac{e\,\check{u}_s \mathring{P}_s}{\kappa_{/\!/} R_{sf}}\,\mathring{\wedge}\,\hat{\mathbb{S}}\,\mathring{\wedge}\left[0 + \frac{D_v \check{u}_{Hs}\beta_{s\perp}}{R_{sf}}\right]\right\}_{@\tau_s = t - R_{sf}/c}$$

±*Twist*

2-Sided Energetics

Energy is transferred between Two Particles, by the symmetric Field / Particle interactions , "dual" between Charge & Helicity .

Positional

$\left(\pm\mathcal{E}_0\frac{R_e}{R_{12}}\mathring{P}\right)$ $\quad \mathcal{E}_{R12}\,\mathring{P}(x) = \boldsymbol{\varphi}(x)\,\mathbb{1}\circ e\,u_Q\langle\mathfrak{R}(x)\rangle_{g1p}$ $\quad \left(\frac{\#e}{R}\circ(\pm e)\,\boldsymbol{\delta}^3\right)$

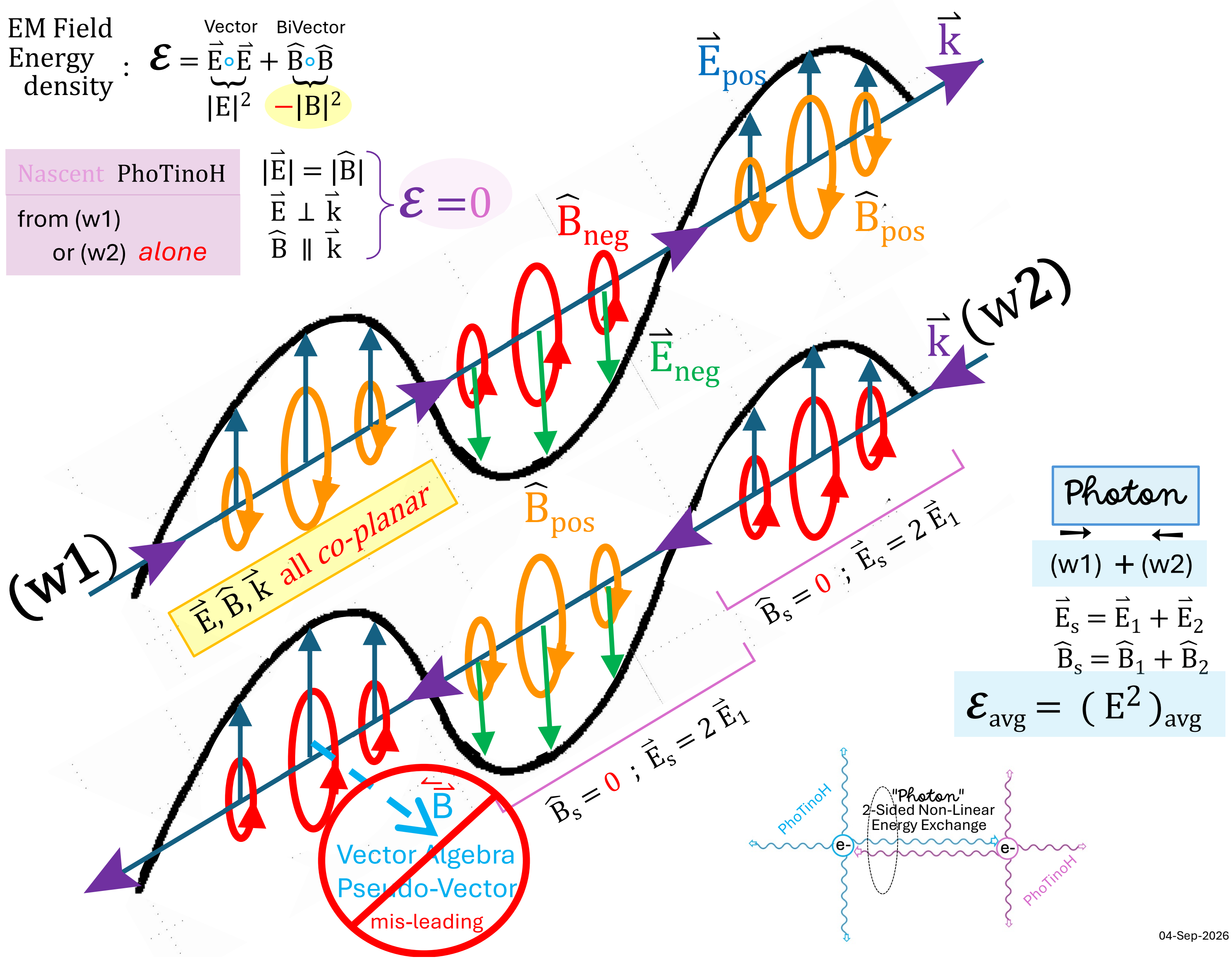
$\vec{\mathcal{F}}(x) = -\vec{\nabla}\,\mathcal{E}_{R12}\,\mathring{P}(x)$

Relative Orientations

$\left(\pm\mathcal{E}_0\frac{R_e}{R_{12}}\mathring{P}\frac{D_v}{R_{12}}\right)$ $\quad \mathcal{E}_{R\theta\theta}\,\mathring{P}(x) = \boldsymbol{\psi}(x)\,\mathcal{T}\circ e\,u_Q\langle u_H\mathfrak{R}(x)\rangle_{g4p}$ $\quad \left(\frac{\#e}{R}\beta_1\circ\beta_2^1\,(\pm e)\,\boldsymbol{\delta}^3\right)$

It Takes Two to Tangle

PhoTinoH | Photon



Each Particle p1,p2 continuously *emanates* outgoing EM Fields w1,w2 ;
these (*linear*) E, B Fields carry no (non-linear) Energy,
eventually being partially *absorbed* by the other Particle,
giving *simultaneous, equal, and opposite* Forces between Particles.