

Geometric Electro-Magnetic Field Equations for Particles with Charge and Dipole Spin

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The geometric and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) Force and Torque interactions between moving Particles with bound Charge and Dipole Spin .

Math Point / Real Particle

The 3D Euclidean geometry is properly described by the **Grassmann extension algebra** with 4 Basis Elements { **3 Vectors, 1 Point** }, here denoted **G3p1** . This generates structures for bound Particles, in addition to lines, planes, and volumes.

This broader geometric algebra can incorporate the two fundamental lengths in Physics :

- (1) the "classical radius of the electron", denoted $R_e \equiv e^2/m_e c^2 = 2.82 \text{ pm} (10^{-15} \text{ m})$, which scales the electric interaction energy between two charged particles; and
- (2) the "Compton wavelength", here denoted $D_v \equiv \lambda_c \equiv \hbar c / m_e c^2 = 386. \text{ pm}$, which scales the magnetic Dipole Spin of a *single* electron .

With this full algebra, **Forces** between 2 or more particles can add to become a **Torque**, and two or more linear **Flows** can add to become a **Circulation** , giving a basis for **Dipole Spin**. This can *not* be expressed in the "2D tangent space" of Clifford Algebra **Cl3** or Vector Algebra.

With similar clarity, the G3p1 algebra describes the motional "transformations" between vector electric $\mathbf{E}$ and bi-vector magnetic $\mathbf{B}$, through 1st order (v / c) motion of an Origin Point, *without* 2nd order space-time algebra effects.

Duality : It Takes Two to Tangle

The triumph of Maxwell's Equations is the description of E-M waves, propagating outward from "charged particles" at fundamental speed c . These are generally viewed as either "light waves", or as "~~Photons~~ with spin", expressing the "wave / particle" duality of modern physics.

In G3p1, the isolated spherical E-M wave ("**PhoTinoH**") emanating outward from a *single* Particle has invariant energy density $\vec{E}^2 + \hat{B}^2 = 0$, because the bi-vector magnetic field squares to negative. That is, 1-sided emanations are "potential", or "nascent", becoming energetically "bipotent" only when coupled to an oppositely propagating PhoTinoH from a second Particle.

That is, , the *superposition of two counter-propagating PhoTinoH* is readily seen to give the inter-particle Force and Torque transfers, incorrectly attributed to a single preternatural *Photon* . Moreover, the symmetry of the two PhoTinoH enables the "equal, opposite, and simultaneous" "Newtonian ideal interaction", despite the finite propagation delays from speed c .

In the 3D algebra of G3p1, the Particle-Field-Particle equations display "CP $\mathcal{H}$ " (rather than CPT) symmetries: 3 geometric Parity "flips" $\check{u}_x \check{u}_y \check{u}_z$; plus particle Charge $\check{u}_Q$; and particle $\mathcal{H}$ elicity $\check{u}_{\mathcal{H}}$. These sufficiently distinguish 4 particles { e⁻, p⁺, e⁺, p⁻ }, and show the scalar field $\phi(x,t)$ to be dual to the scalar field $\psi(x,t)$ describing tri-vector "Twist" .

Moreover, geometric $\mathcal{H}$ elicity shows that the velocity $\vec{\beta}$ of an electron is always perpendicular to the symmetry plane of the Dipole Spin ; and necessarily, $|\beta| > 0$. Here, the dynamics of closely interacting "spins" may shed light on the "quantum of uncertainty", expressed as $\Delta x \gtrsim D_v / \beta_x$.

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Geometric Linear Algebras

Geometric : $\vec{A} \wedge \vec{B} = \vec{A} \circ \vec{B} + \vec{A} \wedge \vec{B}$
Product $\wedge$ extend $\perp$
contract $//$

G3p1 $\equiv$ Grassmann Extension Algebra

3+1 Basis Elements : **3 Vectors** +1 **Point**
or: **3+1** Points

3-Dimensional Affine Space $(\mathcal{A}, \mathcal{V}^{(3)})$ points carries

2D Tangent Space

cl3 $\equiv$ Clifford Geometric Algebra (Hestenes)

3 Basis Elements: **3 Vectors**

G3p1 gives the full Division Algebra *needed* for
geometric Particle/Field interactions in real 3D-Space

Wedge, [Cross], Extension, Protrusion $\wedge$ is $\perp$
Dot, [Scalar], Contraction, Adherence $\circ$ is $//$

$\check{u}^2 = 1 \rightarrow \pm$ flip
 $\check{i}^4 = 1 \rightarrow 90^\circ$ rot

G3p1
 $N_{\text{BasEl}}=4$

Length scaling $\mathbb{L}^0$ $\delta^3 \mathbb{L}^1$ $\delta^3 \mathbb{L}^2$ $\delta^3 \mathbb{L}^3 = \mathbb{L}^0$

$g_0 \#1$ $g_1 \#4$ $g_2 \#6$ $g_3 \#4$ $g_4 \#1 \#16$

$\vec{e}_x = \vec{P}_1 - \vec{P}_0$

$\vec{R}_x = \vec{P}_0 \wedge \vec{e}_x$ $\vec{U}_{yz} = \vec{e}_y \wedge \vec{e}_z$

$\vec{S}_{yz} = \vec{P}_0 \wedge \vec{U}_{yz}$

$\vec{I}_4 = \vec{P}_0 \wedge \vec{e}_x \wedge \vec{e}_y \wedge \vec{e}_z$

$\vec{P}_0 + \vec{e}_{0i} = \vec{P}_i$

$\vec{R}_{0i} + \vec{U}_{ji} = \vec{R}_{ji}$

$\vec{S}_{0jk} + \vec{T}_{ijk} = \vec{S}_{ijk}$

$\vec{E}_1 + \vec{E}_2 = \vec{B}_1 + \vec{E}_3$

$\vec{F}_1 + \vec{F}_2 = \vec{T}_1 + \vec{F}_3$

$\vec{J}$ Current
 $\vec{E}$ Electric Field
 $\vec{B}$ Magnetic Circulation
 $\vec{T}$ Torque
 $\vec{S}$ Spin
 $\vec{R}$ Ring Dipole of Spin

$\pm$ Pressure
Point
Force on Particle
Electric Emission
Magnetic Circulation
 $\pm$ Twist
 $\pm$ Helicity

3D 4π

E&M φ

3D Tetra-Hedron
Helicity $\check{u}_H$

Tangent Space

cl3
 $N_{\text{BasEl}}=3$

Pauli $S^{1/2}$
Quaternions = g_0, g_2

Gibbs Vec Alg

Vector

Pseudo-Vec

$\vec{E}$ $\vec{B}$

$\vec{I}_3 = \vec{e}_x \wedge \vec{e}_y \wedge \vec{e}_z$

$\check{i}$

$\check{u}_x \check{u}_y \check{u}_z$

$\check{u}_0 \check{u}_x \check{u}_y$

$\check{u}_z \pm \check{u}_0$

G2p1
 $N_{\text{BasEl}}=3$

$g_0 \#1$ $g_1 \#3$ $g_2 \#3$ $g_3 \#1$

$s > 0$

$\vec{P}_1 - \vec{P}_0 = \vec{e}_x$
 $\vec{P}_2 - \vec{P}_0 = \vec{e}_y$

$\vec{R}_y = \vec{P}_0 \wedge \vec{e}_y$

$\vec{U}_{xy} = \vec{e}_x \wedge \vec{e}_y$

$\vec{I}_3 = \vec{P}_0 \wedge \vec{e}_x \wedge \vec{e}_y$

$\check{i}$

$\vec{P}_0 \wedge \vec{e}_x + \vec{e}_y \wedge \vec{e}_x = \vec{P}_2 \wedge \vec{e}_x$

$\check{u}_0 \check{u}_x \check{u}_y$

$\check{u}_z \pm \check{u}_0$

2D 2π

2D Simplex

Tangent Space

cl2
 $N_{\text{BasEl}}=2$

Complex: $\mathbb{Z} = g_0, g_2$

$\vec{e}_x$ $\vec{e}_y$

$\vec{I}_2 = \vec{e}_x \wedge \vec{e}_y$

$\check{u}_x \check{u}_y$

$\check{u}_0 \check{u}_x$

$\check{u}_y \pm \check{u}_0$

G1p1
 $N_{\text{BasEl}}=2$

Length Scale

$\vec{P}_1 - \vec{P}_0 = \vec{e}_x$

$\vec{P}_0 + \vec{e}_x = \vec{P}_1$

$\vec{I}_2 = \vec{P}_0 \wedge \vec{e}_x$

$\vec{P}_1 \circ \vec{P}_1 = 1$

$0\vec{e}_x \circ 0\vec{e}_x = 0$
ZeroVector $\neq$ Point

$\check{u}_0 \check{u}_x$

$\check{u}_y \pm \check{u}_0$

1D 1

1D Simplex

Tangent Space

G0p1

$\check{u}_A$ Existence

$A = \begin{cases} -1 & \text{No / Out / Tails} \\ 1 & \text{Yes / In / Heads} \end{cases}$ Dual

G0pm

$(A, B \dots \text{independent})$
[no geometric carries]

Boolean (Sets)

$\check{u} = \sqrt[2]{1} = \{1, -1\}$ #2

Not $\wedge$ or $\circ$ and

DeMorgan's

$\overline{A \wedge B} = \overline{A} \circ \overline{B}$ #2^m

$\overline{A \circ B} = \overline{A} \wedge \overline{B}$

Maxwell / Lorentz Equations with Charge and Dipole Spin

Traditional 2D Fluid Algebras, no Time Delay

Cl3 Geometric Algebra
Clifford / Hestenes
2D Tangent Space

Std. Vector Algebra
Gibbs / Heaviside
1D+

• Dot
∧ Wedge

• Dot
× Cross

g0 $\nabla \circ \vec{E} = \Leftrightarrow \nabla \circ \vec{E} = 4\pi (\rho_+ - \rho_-)$

g1 $-\nabla \circ \vec{B} = \Leftrightarrow \nabla \times \vec{B} = \frac{\partial}{\partial ct} \vec{E} + 4\pi \vec{J}/c$

g2 $\nabla \wedge \vec{E} = \Leftrightarrow \nabla \times \vec{E} = -\frac{\partial}{\partial ct} \vec{B}$

g3 $\nabla \wedge \vec{B} = \Leftrightarrow \nabla \circ \vec{B} = 0$

bi-vec pseudo-vector

2-fluids $\vec{J}/c$

$\mathcal{Q}$ $\mathcal{T}$ Twist, Helicity

Geometric Product

$$A \frown B \equiv A \wedge B + A \circ B$$

∧ Orthogonal Extension
○ Parallel Contraction

Cl3 and G3p1 build graded structures with the Geometric Product.
G3p1 builds up to Vorticity.

Units

$\mathbb{L}$ [meter]
 $\mathbb{T} \equiv \mathbb{L}/c * 299.79 \times 10^6$ [sec]
 $\mathcal{E}_0 \equiv m_e c^2 \sim 511.$ [keV]

Length scales of E & M

$\mathcal{R}_e = \frac{e^2}{\mathcal{E}_0} \sim 2.82 \times 10^{-15}$ m
 $\mathcal{D}_v = \frac{\hbar c}{\mathcal{E}_0} \sim 386. \times 10^{-15}$ m

Standard "Gibbs Vector Algebra" becomes *inconsistent* at pseudo-vector, and can not represent 2 fluids .
 Cl3 has 3 Basis Vectors, representing a consistent 2D Tangent Space, but *cannot* represent 3D Particles or Spin.

G3p1 linear algebra connects 3-D Fields to Particle Sources with Charge and 3-D Dipole Spin.

1-Sided Linear Algebra, giving "Nascent" Fields from Source Particles $\mathcal{P}_s$.

CPH Symmetry

$$\mathfrak{u} = \{+, -\}$$

Geometric Parity $\mathfrak{u}_0 \mathfrak{u}_x \mathfrak{u}_y \mathfrak{u}_z$

	Bound Particles			
	e-	p+	e+	p-
Charge $\mathfrak{u}_Q$	-	+	+	-
Helicity $\mathfrak{u}_H$	+	+	-	-

g0 $\left(\frac{\#e}{R} \mathbb{1} \right)$

g1p $\left(\frac{1}{LL} \frac{\#e}{R} \right)$

g1 $\left(\frac{\#e}{RL} \right)$

g1 $\left(\frac{1}{ct} \frac{\#e}{RL} \right)$

g2 $\left(\frac{1}{ct} \frac{\#e}{R} \beta \right)$

g2 $\left(\frac{\#e}{R} \beta \right)$

g4p $\left(\frac{1}{LL} \frac{\#e}{R} \beta \right)$

g3 $\left(\frac{\#e}{R} \beta \right)$

Nascent Field $\Leftrightarrow$ **Emanation** **Fluidize** **Source Particles** $@(x_s, \tau_s)$ **Monopole Charge** **Dipole Charge** **Distant Time Delay**

$$\boldsymbol{\varphi}(x_0, t) \mathbb{1} \leftarrow \int d^3x \sum_{s \neq f} \left\{ \frac{e \mathfrak{u}_s \mathcal{P}_s}{\kappa_{//} R_{sf}} \mathbb{1} \left(1 + \frac{\vec{d}_s \circ \vec{r}_{sf}}{R_{sf}} \right) \right\} @ \tau_s = t - R_{sf}/c$$

±Pressure

$$\frac{\nabla^2}{4\pi} \boldsymbol{\varphi}(x_s, t) \mathbb{1} = \frac{\vec{\nabla}}{4\pi} \circ \vec{\mathcal{E}}(\sim x_s) \Leftrightarrow e \mathfrak{u}_s \langle \mathcal{R}_s \rangle_{g1p}$$

$$\vec{\mathcal{E}}(x) = -\vec{\nabla} \wedge \boldsymbol{\varphi}(x)$$

$$\left\{ \begin{array}{l} \frac{\partial}{\partial ct} \vec{\mathcal{E}}(x) = \frac{\vec{\nabla}}{4\pi} \circ \vec{\mathcal{B}}(x) \\ \frac{\partial}{\partial ct} \vec{\mathcal{B}}(x) = \vec{\nabla} \wedge \vec{\mathcal{E}}(x) \end{array} \right.$$

Vacuum All (x,t)

$$\vec{\mathcal{B}}(x) = -\vec{\nabla} \circ \boldsymbol{\psi}(x) \tilde{\mathcal{T}}$$

$$\frac{\nabla^2}{4\pi} \boldsymbol{\psi}(x_s, t) \tilde{\mathcal{T}} = \vec{\nabla} \wedge \vec{\mathcal{B}}(\sim x_s) \Leftrightarrow e \mathfrak{u}_s \langle \mathfrak{u}_H \mathcal{R}_s \rangle_{g4p}$$

Nascent Field $\Leftrightarrow$ **Emanation** **Fluidize** **Source Particles** $@(x_s, \tau_s)$ **No Monopole Spin** **Dipole of Spin** **Distant Time Delay**

$$\boldsymbol{\psi}(x_0, t) \tilde{\mathcal{T}} \leftarrow \int d^3x \sum_{s \neq f} \left\{ \frac{e \mathfrak{u}_s \mathcal{P}_s}{\kappa_{//} R_{sf}} \hat{\mathcal{S}} \left[0 + \frac{\mathcal{D}_v \mathfrak{u}_{HS} \vec{\beta}_{s\perp}}{R_{sf}} \right] \right\} @ \tau_s = t - R_{sf}/c$$

±Twist

$$\vec{r}_{sf} = \mathcal{P}_s - \mathcal{P}_f$$

$R = |\vec{r}_{sf}|$

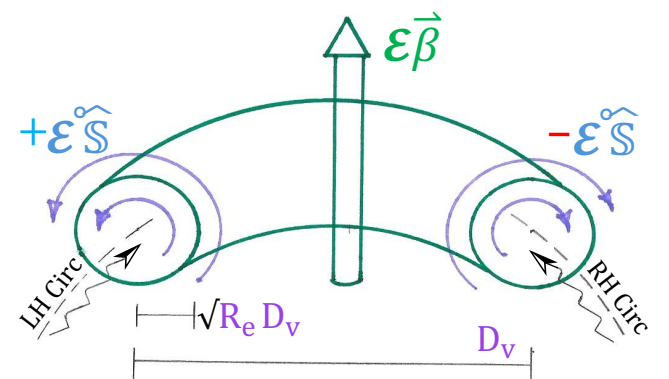
$$\vec{\beta} = \frac{\partial}{\partial ct} (\mathcal{P}_s - \mathcal{P}_0)$$

$$\vec{\beta}_{//} = (\vec{\beta} \circ \vec{r}) \frac{\vec{r}}{r}$$

$$\vec{\beta}_{\perp} = (\vec{\beta} \wedge \vec{r}) \circ \frac{\vec{r}}{r}$$

$$\kappa_{//} = 1 - \vec{\beta}_{//} \circ \vec{r}$$

Toroidal Ring of Charge



Toroidal Ring of Dipole Spin

Electron Geometry

2-Sided Energetics

Energy is transferred between Two Particles, by the symmetric Field / Particle interactions , "dual" between Charge & Helicity .

Positional $\left(\pm \frac{\mathcal{R}_e}{R_{12}} \mathcal{P} \right)$ $\mathcal{E}_{R12} \mathcal{P}(x) = \boldsymbol{\varphi}(x) \mathbb{1} \circ e u_Q \langle \mathcal{R}(x) \rangle_{g1p}$

Relative Orientations $\left(\pm \frac{\mathcal{R}_e}{R_{12}} \mathcal{P} \frac{\mathcal{D}_v}{R_{12}} \right)$ $\mathcal{E}_{R0\theta} \mathcal{P}(x) = \boldsymbol{\psi}(x) \tilde{\mathcal{T}} \circ e u_Q \langle \mathfrak{u}_H \mathcal{R}(x) \rangle_{g4p}$

$\left(\frac{\#e}{R} \beta_1 \circ \beta_2^1 (\pm e) \delta^3 \right)$

$$\vec{\mathcal{F}}(x) = -\vec{\nabla} \mathcal{E}_{R12} \mathcal{P}(x)$$

It Takes **Two** to Tangle

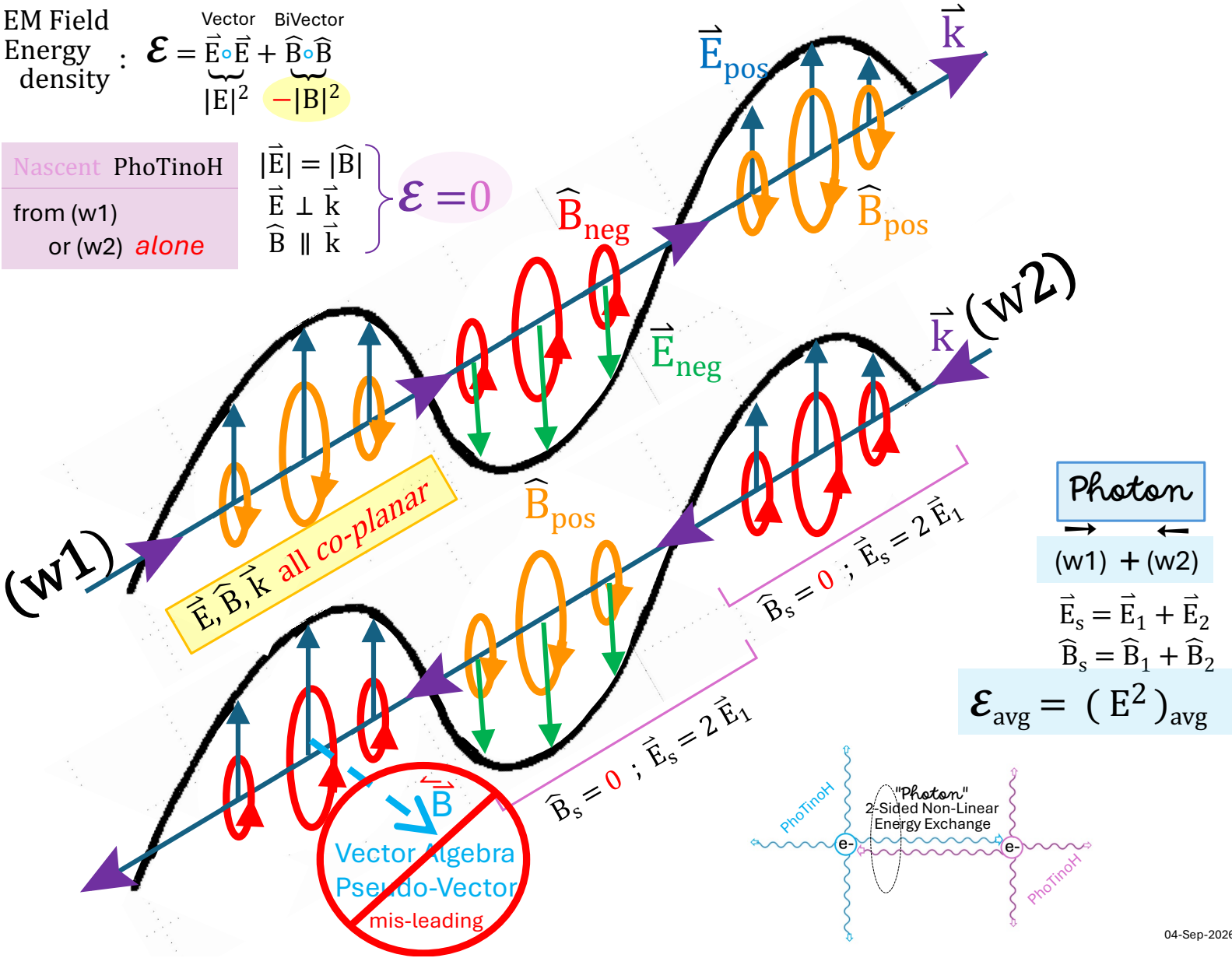
PhoTinoH

Photon

EM Field Energy density : $\mathcal{E} = \underbrace{\vec{E} \cdot \vec{E}}_{|\vec{E}|^2} + \underbrace{\vec{B} \cdot \vec{B}}_{-|\vec{B}|^2}$

Nascent PhoTinoH from (w1) or (w2) **alone**

$$\left. \begin{array}{l} |\vec{E}| = |\vec{B}| \\ \vec{E} \perp \vec{k} \\ \vec{B} \parallel \vec{k} \end{array} \right\} \mathcal{E} = 0$$



Each Particle **p1**, **p2** continuously *emanates* outgoing EM Fields **w1**, **w2** ; these (*linear*) E, B Fields carry no (non-linear) Energy, eventually being partially *absorbed* by the other Particle, giving *simultaneous, equal, and opposite Forces* between Particles.

"PhoTinoH" **w1**, **w2**

