

Geometric Electro-Magnetic Field Equations for Particles with Charge, Spin and Helicity

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The geometric and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) Force and Forque interactions between moving particles with Charge and Helicity (a.k.a. "spin").

Math Point => Real Particle

The algebra is the **4-level Grassmann linear algebra** describing 3D Euclidean space, with 4 Basis Elements { **3 Vectors, 1 Point** }, generating structures for bound Particles, in addition to lines, planes, and volumes. Here, this is denoted as **G3p1**.

This geometric algebra incorporates the two fundamental lengths in E-M :

- (1) the "classical radius of the electron" $R_e \equiv e^2/m_e c^2 = 2.82 \text{ pm}$ (10^{-15} m), which scales the electric interaction energy between *two* charged particles; and
- (2) the "Compton wavelength", here denoted $D_v \equiv \hbar c / m_e c^2 = 386. \text{ pm}$, which scales the magnetic Helicity of a *single* electron or proton.

With this full algebra, **Forces** from 2 or more particles can add to become a **Torque**, and two or more linear **Flows** can add to become a **Circulation**. This can *not* be expressed in the "2D tangent space" of Clifford Algebra **Cl3**, nor in standard (Gibbs) Vector Algebra.

With similar clarity, the **G3p1** algebra describes the motional "transformations" between vector electric $\vec{E}$ and bi-vector magnetic $\vec{B}$, through 1st order ($\vec{v} / c$) motion of an Origin Point, *without* 2nd order space-time algebra effects.

Duality : It Takes Two to Tangle

The triumph of Maxwell's Equations is the description of EM waves, propagating outward from particles at fundamental speed c . These are generally viewed as either "light waves", or as particle-like *Photons*, expressing the "wave / particle" duality of modern physics.

In **G3p1**, the spherical EM wave ("Photino") emanating outward from a *single* particle is clearly seen to be "nilpotent", with invariant energy density $\vec{E}^2 + \vec{B}^2 = 0$. That is, 1-sided emanations are "potential", or "nascent", until coupled to a 2nd wave or particle. Then, multiple Photinos will add, subtract, and diffract as waves.

In contrast, the *superposition* of *two counter-propagating Photinos* is readily seen to give the inter-particle Force and Torque transfers generally attributed to a single preternatural *Photon*. Moreover, the symmetry of the two Photinos ensures the "equal, opposite, and simultaneous" Newtonian ideal, despite the finite propagation delay from speed c .

In the 3D algebra of **G3p1**, the Field/Particle equations display "PC $\mathcal{H}$ " symmetries: 4 geometric Parity $\vec{u}_0 \vec{u}_x \vec{u}_y \vec{u}_z$; plus particle Charge $\vec{u}_Q$; plus particle $\mathcal{H}$ elicity $\vec{u}_H$. Here, $\mathcal{H}$ elicity replaces the (hypothetical) Dirac Magnetic Monopole. These 6 symmetries sufficiently distinguish particles { e-, p+, e+, p- }; and the Time parameter has no "Time-reversal symmetry" as in CPT.

Moreover, geometric $\mathcal{H}$ elicity demands that the velocity β of an electron is always perpendicular to the "spin" plane $\hat{M}$; and necessarily, $|\beta| > 0$. Understanding the dynamics of closely interacting "spins" may shed light on the "quantum uncertainty", expressed as $\Delta\beta \Delta x \gtrsim D_v$.

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Geometric Linear Algebras

G3p1 $\equiv$ Grassmann Extension Algebra

3+1 Basis Elements : **3 Vectors +1 Point**
or: **3+1 Points**

3-Dimensional
Affine Space

points carries
($\mathcal{A}$, $\mathcal{V}^{(3)}$)

4 π

2D Tangent Space

Cl3 $\equiv$ Clifford Geometric Algebra (Hestenes)

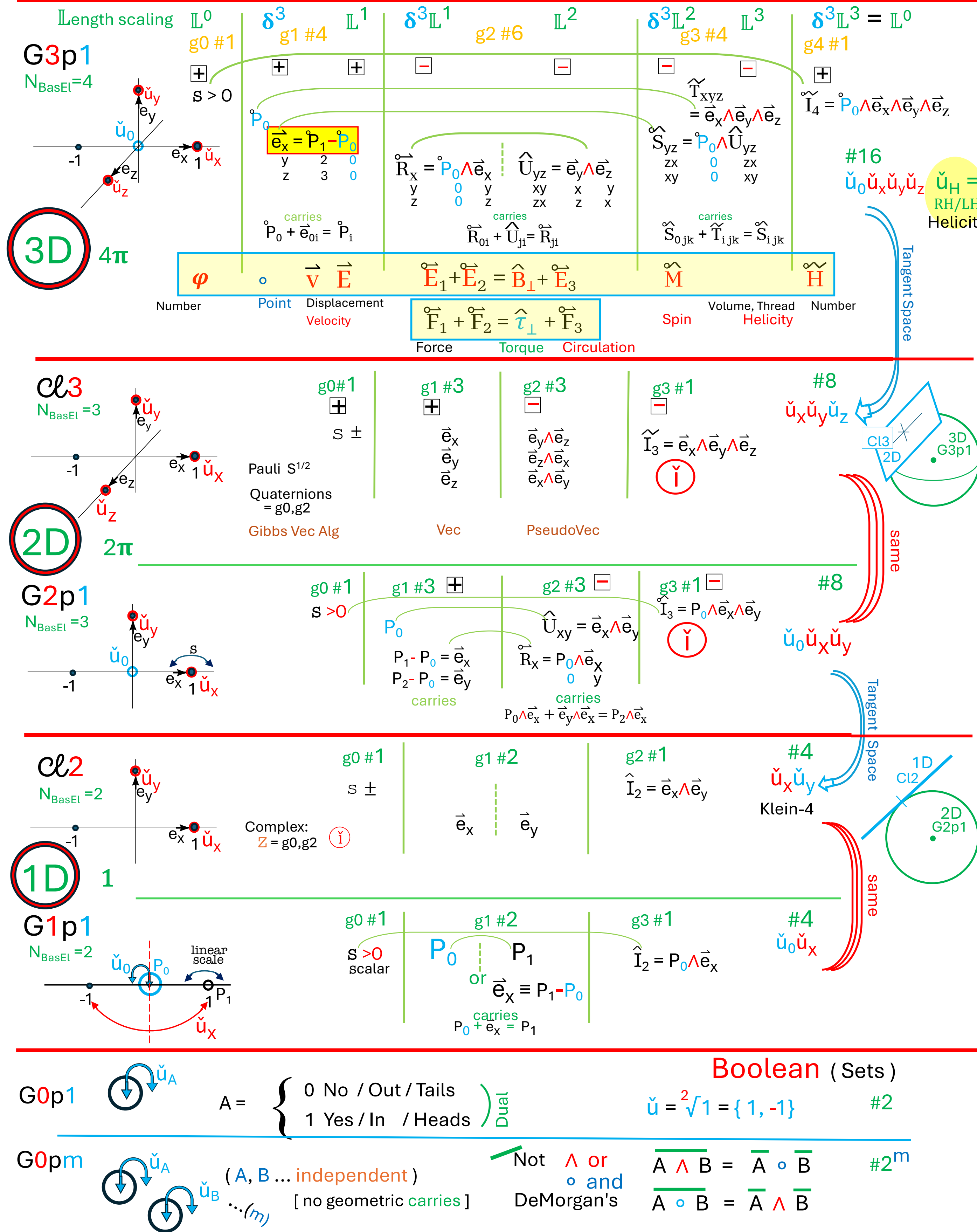
3 Basis Elements: **3 Vectors**

G3p1 gives full structures *needed* for geometric Particle/Field interactions in real 3D-Space

Wedge, [Cross],
Extension, Protrusion $\wedge$ is $\perp$
Dot, [Scalar],
Contraction, Adherence $\circ$ is $\parallel$

$\vec{u} = \sqrt[2]{1}$ = flip
 $\vec{i} = \sqrt[4]{1} = 90^\circ \text{rot}$

Geometric Product $\tilde{\lambda}$: $\vec{A} \wedge \vec{B} = \vec{A} \circ \vec{B} + \vec{A} \wedge \vec{B}$
extend $\perp$
contract $\parallel$



Maxwell / Lorentz Equations for Charge and "Spin"

Traditional 2D Fluid Algebras

Cl3 Geometric Algebra
Clifford / Hestenes
2D Tangent Space

$\circ$ Dot
 $\wedge$ Wedge

Std. Vector Algebra
Gibbs / Heaviside
1D+

$\cdot$ Dot
 $\times$ Cross

Geometric Product

$$A \tilde{\lambda} B \equiv A \wedge B + A \circ B$$

$\wedge$ Orthogonal Extension

$\circ$ Parallel Contraction

Cl3 and **G3p1** build graded structures with the Geometric Product.
G3p1 builds up more "entanglement".

$$\begin{aligned} g_0 \quad \nabla \circ \vec{E} &\Leftrightarrow \nabla \circ \vec{E} = 4\pi (\rho_+ - \rho_-) \\ g_1 \quad -\nabla \circ \vec{B} &\Leftrightarrow \nabla \times \vec{B} = \frac{\partial}{\partial ct} \vec{E} + 4\pi \vec{J}_c \\ g_2 \quad \nabla \wedge \vec{E} &\Leftrightarrow \nabla \times \vec{E} + \frac{\partial}{\partial ct} \vec{B} = 0 \\ g_3 \quad \nabla \wedge \vec{B} &\Leftrightarrow \nabla \circ \vec{B} = 0 \end{aligned}$$

bi-vec pseudo-vector

Standard Vector "Algebra" becomes inconsistent at 1D+ pseudo-vector and can not represent 2 fluids.
Cl3 is a consistent Linear Algebra, with 3 Basis Vectors, giving for a 2D Tangent Space, but cannot represent Particles.

G3p1 linear algebra connects 3-D Fields to Particle Sources with Charge and 3-D Helicity

1-Sided "Nascent" Potentiality

$$\begin{aligned} \text{dual} \quad \left\{ \begin{aligned} g_0 \quad \varphi(x_f, t) &= e \int d\tilde{x} \tilde{\lambda} \sum_s \left\{ \frac{\vec{u}_s \cdot \vec{p}_s}{R} \tilde{\lambda} \left(1 + \frac{\vec{\beta} \cdot \vec{r}}{R} + \frac{\vec{d} \cdot \vec{r}}{R} \right) \right\} @_{\tau_s = t - R/c} \\ g_1 \quad \vec{E} &\equiv -\vec{\nabla} \wedge \varphi \\ g_2 \quad \vec{\nabla} \circ \vec{E} &= -\frac{\nabla^2}{4\pi} \varphi = e \sum_s \left\{ \frac{\vec{u}_s \cdot \vec{p}_s}{R} \tilde{\lambda} \left(1 + 2\beta_{\parallel} + 2d_{\parallel} \right) \right\} @_{\tau_s} \\ g_3 \quad \vec{\nabla} \wedge \vec{B} &= \frac{\nabla^2}{4\pi} \tilde{\mathcal{H}} = e \sum_s \left\{ \frac{\vec{u}_s \cdot \vec{p}_s}{R} \tilde{\lambda} \left(0 + 2\vec{\beta} \wedge \vec{r} \cdot \vec{T} + 2\vec{\beta} \wedge \hat{M} \right) \right\} @_{\tau_s} \end{aligned} \right. \\ \text{Helicity} \quad \tilde{\mathcal{H}}(x_f, t) &= e \int d\tilde{x} \tilde{\lambda} \sum_s \left\{ \frac{\vec{u}_s \cdot \vec{p}_s}{R} \tilde{\lambda} \left(0 + \frac{\vec{\beta} \wedge \vec{r} \cdot \vec{T}}{R} + \frac{\vec{\beta} \wedge \hat{M}}{R} \right) \right\} @_{\tau_s} \end{aligned}$$

Source Points @ (x_s, t_s)

Charge Moving Charge Dipole Charge Time Delay

$\vec{r}_{sf} = \vec{p}_s - \vec{p}_f$
 $R = |\vec{r}_{sf}|$
 $\vec{\beta} = \frac{\partial}{\partial ct} (\vec{p}_s - \vec{p}_0)$

$\vec{d}_s \equiv \vec{p}_s^+ - \vec{p}_s^- \neq 0$

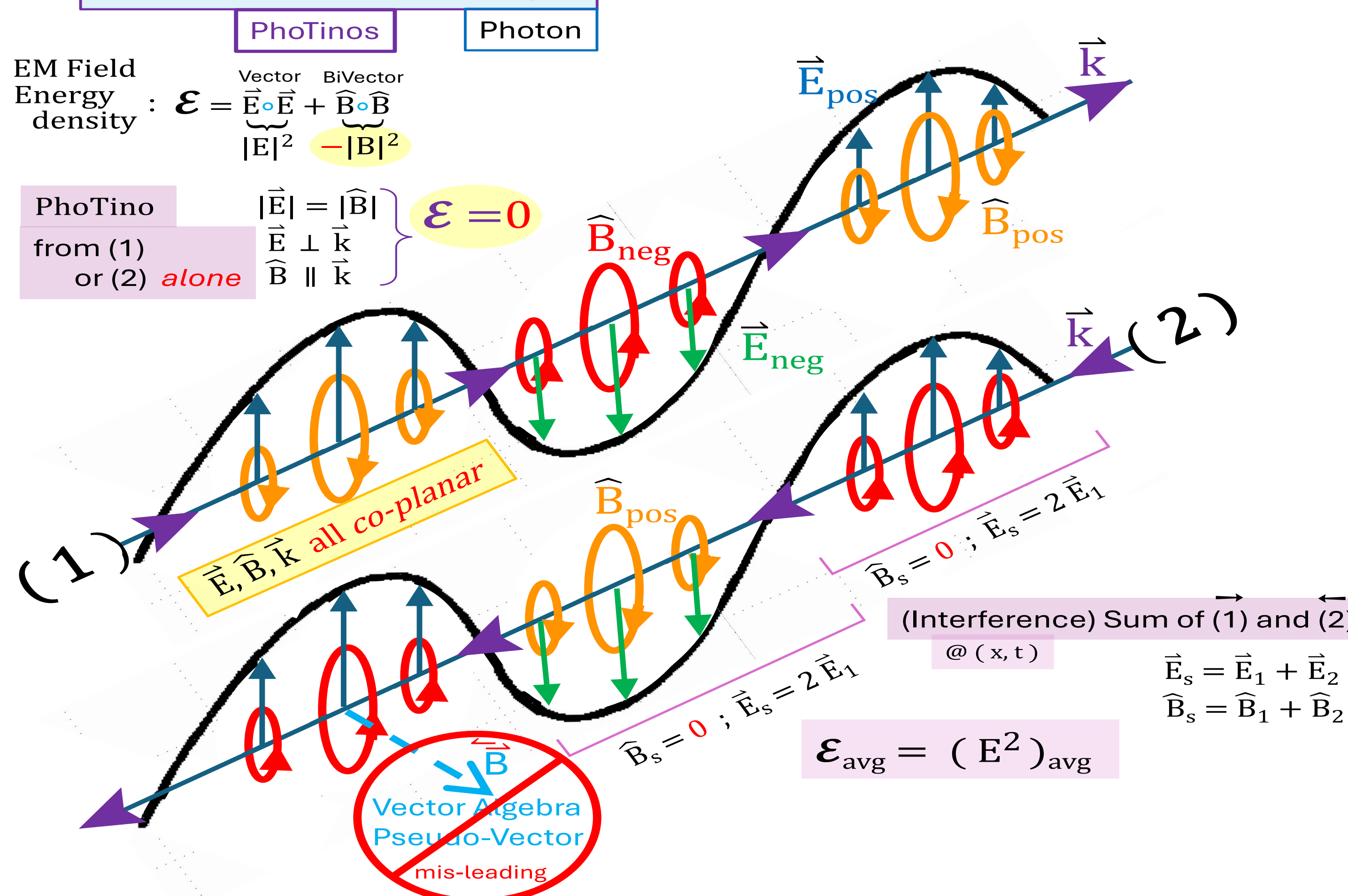
$\hat{M} = \frac{\vec{u}_H D_v}{2} (1 - \beta^2) (\vec{\beta} \circ \vec{T})$
Spin
i.e. $\hat{M} \perp \vec{\beta}$

2-Sided Energetics

Energy is transferred between 2 Bound Particles by the symmetric Field / Particle interactions, "dual" between Charge / Helicity.

$$\begin{aligned} \text{Energy} \quad \vec{\mathcal{E}} \cdot \vec{p} &= q_f p_f \varphi^{(x,t)} + e p \wedge \vec{d} \circ \vec{E}^{(x,t)} - q p \wedge \hat{M} \circ \vec{B}^{(x,t)} \\ \text{Force} \quad \vec{\mathcal{F}} &= q p \wedge \vec{E}^{(x,t)} - q p \wedge \vec{\beta} \circ \vec{B}^{(x,t)} + e p \wedge (\vec{d} \circ \vec{\nabla}) \vec{E}^{(x,t)} - q p \wedge \hat{M} \circ (\vec{\nabla} \circ \vec{B}^{(x,t)}) \\ \text{Torque} \quad \vec{\mathcal{T}} &= + e p \wedge \vec{d} \wedge \vec{E}^{(x,t)} - q p \wedge \hat{M} \wedge \vec{B}^{(x,t)} \end{aligned}$$

It Takes Two to Tangle



Particles (1),(2) emanate "NilPotent" spherical waves w1,w2 ; waves w1,w2 give simultaneous Forces on particles (1),(2).

