Geometric Electro-Magnetic Field Equations for Particles with Charge, Spin and Helicity

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The geometric and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) Force and Forque interactions between moving particles with Charge and Helicity (a.k.a. "spin") .

Math Point => Real Particle

The algebra is the **4-level Grassmann linear algebra** describing 3D Euclidean space, with 4 Basis Elements **{ 3 Vectors, 1 Point }**, generating structures for bound Particles, in addition to lines, planes, and volumes. Here, this is denoted as **G3p1** .

This geometric algebra incorporates the two fundamental lengths in E-M :
(1) the "classical radius of the electron" $\mathbf{R_e} \equiv e^2/m_e c^2 = 2.82$ pm (10^{-15} m), which scales the electric interaction energy between *two* charged particles; and
(2) the "Compton wavelength", here denoted $\mathbf{D_v} \equiv \hbar c / m_e c^2 = 386.$ pm, which scales the magnetic Helicity of a *single* electron or proton.

With this full algebra, **Forces** from 2 or more particles can add to become a **Torque**, and two or more linear **Flows** can add to become a **Circulation** . This can *not* be expressed in the "2D tangent space" of Clifford Algebra Cℓ3 , nor in standard (Gibbs) Vector Algebra.

With similar clarity, the G3p1 algebra describes the motional "transformations" between vector electric $\vec{\mathrm{E}}$ and bi-vector magnetic $\widehat{\mathrm{B}}$, through 1st order ($\vec{\mathrm{v}}$ / c) motion of an Origin Point, *without* 2nd order space-time algebra effects.

Duality : It Takes Two to Tangle

The triumph of Maxwell's Equations is the description of EM waves, propagating outward from particles at fundamental speed **c** . These are generally viewed as either "light waves", or as particle-like *Photons*, expressing the "wave / particle" duality of modern physics.

In G3p1, the spherical EM wave ("Photino") emanating outward from a *single* particle is clearly seen to be "nilpotent", with invariant energy densiy $\vec{\mathrm{E}}^2 + \widehat{\mathrm{B}}^2 = 0$. That is, 1-sided emanations are "potential", or "nascent", until coupled to a 2nd wave or particle. Then, multple Photinos will add, subtract, and diffract as waves.

In contrast, the *superposition of two counter-propagating Photinos* is readily seen to give the inter-particle Force and Torque transfers generally attributed to a single preternatural *Photon*. Moreover, the symmetry of the two Photinos ensures the "equal, opposite, and simultaneous" Newtonian ideal, despite the finite propagation delaly from speed **c** .

In the 3D algebra of G3p1, the Field/Particle equations display "PC$\mathcal{H}$" symmetries: 4 geometric Parity $\check{u}_0\check{u}_x\check{u}_y\check{u}_z$; plus particle Charge $\check{u}_Q$; plus particle $\mathcal{H}$elicity $\check{u}_H$. Here, $\mathcal{H}$elicity replaces the (hypothetical) Dirac Magnetic Monopole. These 6 symmetries sufficiently distinguish particles { e-,p+, e+,p- } ; and the Time parameter has no "Time-reversal symmetry" as in CPT.

Moreover, geometric $\mathcal{H}$elicity demands that the velocity β of an electron is always perpendicular to the "spin" plane $\widehat{\mathrm{M}}$; and necessarily, $|\beta| > 0$. Understanding the dynamics of closely interacting "spins" may shed light on the "quantum uncertainty", expressed as $\Delta\beta\, \Delta\mathrm{x} \gtrsim \mathrm{D_v}$.

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Geometric Linear Algebras

Geometric Product $\overset{\wedge}{\circ}$: $\vec{A}\overset{\wedge}{\circ}\vec{B} = \vec{A}\circ\vec{B} + \vec{A}\wedge\vec{B}$ (extend ⊥; contract //)

G3p1 ≡ Grassmann Extension Algebra
3+1 Basis Elements : **3 Vectors +1 Point**
or : 3+1 Points

3-Dimensional Affine Space — points, carries: $(\mathcal{A}, \mathcal{V}^{(3)})$ 4π → 2π 2D Tangent Space

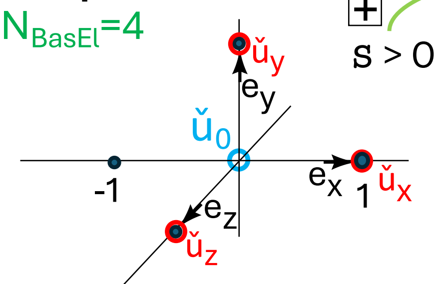
G3p1 gives full structures *needed* for geometric Particle/Field interactions in real 3D-Space

$\mathcal{C}\ell 3$ ≡ Clifford Geometric Algebra (Hestenes)
3 Basis Elements: 3 **Vectors**

Wedge, [Cross], Extension, Protrusion — ∧ is ⊥
Dot, [Scalar], Contraction, Adherence — ∘ is //

$\check{u} = \sqrt[2]{1}$ = flip
$\check{\imath} = \sqrt[4]{1}$ = 90°rot

G3p1 — $N_{BasEl}=4$ — 3D 4π



Length scaling: $\mathbb{L}^0$ | δ^3 $\mathbb{L}^1$ | $\delta^3\mathbb{L}^1$ $\mathbb{L}^2$ | $\delta^3\mathbb{L}^2$ $\mathbb{L}^3$ | $\delta^3\mathbb{L}^3 = \mathbb{L}^0$

- **g0 #1** ⊞ : s > 0
- **g1 #4** ⊞ ⊞ : $\mathring{P}_0$; $\vec{e}_x = \mathring{P}_1 - \mathring{P}_0$ (y: 2, 0; z: 3, 0); carries: $\mathring{P}_0 + \vec{e}_{0i} = \mathring{P}_i$
- **g2 #6** ⊟ ⊟ : $\overleftrightarrow{R}_x = \mathring{P}_0\wedge\vec{e}_x$ (0 y; 0 z) ⋮ $\hat{U}_{yz} = \vec{e}_y\wedge\vec{e}_z$ (xy x y; zx z x); carries: $\overleftrightarrow{R}_{0i} + \hat{U}_{ji} = \overleftrightarrow{R}_{ji}$
- **g3 #4** ⊟ ⊟ : $\widehat{S}_{yz} = \mathring{P}_0\wedge\hat{U}_{yz}$ (zx 0 zx; xy 0 xy); $\widetilde{T}_{xyz} = \vec{e}_x\wedge\vec{e}_y\wedge\vec{e}_z$; carries: $\widehat{S}_{0jk} + \widetilde{T}_{ijk} = \widehat{S}_{ijk}$
- **g4 #1** ⊞ : $\widetilde{I}_4 = \mathring{P}_0\wedge\vec{e}_x\wedge\vec{e}_y\wedge\vec{e}_z$

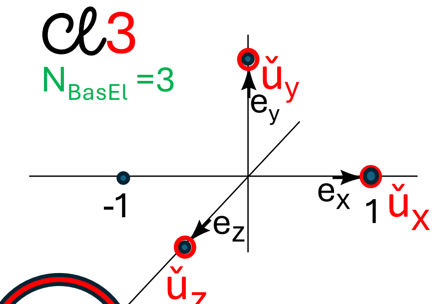
#16 $\check{u}_0\check{u}_x\check{u}_y\check{u}_z$; $\check{u}_H$ = RH/LH Helicity

φ	∘	$\vec{v}$ $\vec{E}$	$\overleftrightarrow{E}_1 + \overleftrightarrow{E}_2 = \hat{B}_\perp + \overleftrightarrow{E}_3$	$\widehat{M}$	$\widetilde{H}$
Number	Point	Displacement, Velocity		Spin	Volume, Thread, Helicity; Number

$\overleftrightarrow{F}_1 + \overleftrightarrow{F}_2 = \hat{\tau}_\perp + \overleftrightarrow{F}_3$ — Force, Torque, Circulation

Tangent Space

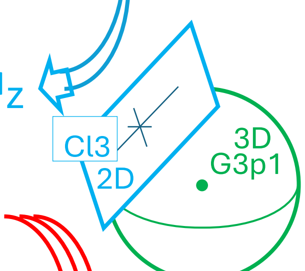
$\mathcal{C}\ell 3$ — $N_{BasEl}=3$ — 2D 2π



- **g0#1** ⊞ : s ±
- **g1 #3** ⊞ : $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ — Vec
- **g2 #3** ⊟ : $\vec{e}_y\wedge\vec{e}_z$, $\vec{e}_z\wedge\vec{e}_x$, $\vec{e}_x\wedge\vec{e}_y$ — PseudoVec
- **g3 #1** ⊟ : $\widetilde{I}_3 = \vec{e}_x\wedge\vec{e}_y\wedge\vec{e}_z$; ǐ

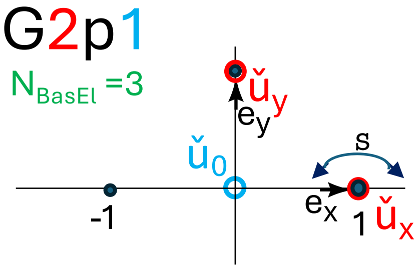
Pauli $S^{1/2}$
Quaternions = g0,g2
Gibbs Vec Alg

#8 $\check{u}_x\check{u}_y\check{u}_z$



same

G2p1 — $N_{BasEl}=3$

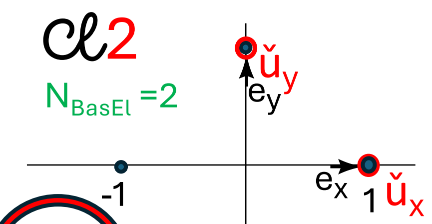


- **g0 #1** : s >0
- **g1 #3** ⊞ : P_0 ; $P_1 - P_0 = \vec{e}_x$; $P_2 - P_0 = \vec{e}_y$; carries
- **g2 #3** ⊟ : $\hat{U}_{xy} = \vec{e}_x\wedge\vec{e}_y$; $\overleftrightarrow{R}_x = P_0\wedge\vec{e}_x$ (0 y); carries: $P_0\wedge\vec{e}_x + \vec{e}_y\wedge\vec{e}_x = P_2\wedge\vec{e}_x$
- **g3 #1** ⊟ : $\widetilde{I}_3 = P_0\wedge\vec{e}_x\wedge\vec{e}_y$; ǐ

#8 $\check{u}_0\check{u}_x\check{u}_y$

Tangent Space

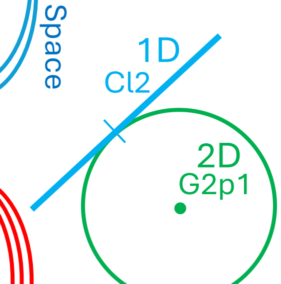
$\mathcal{C}\ell 2$ — $N_{BasEl}=2$ — 1D 1



- **g0 #1** : s ±
- **g1 #2** : $\vec{e}_x$ ⋮ $\vec{e}_y$
- **g2 #1** : $\hat{I}_2 = \vec{e}_x\wedge\vec{e}_y$

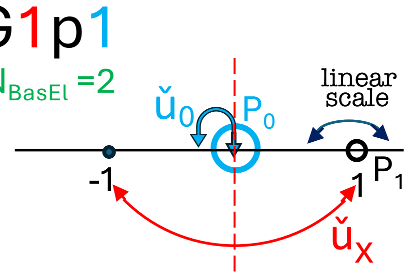
Complex: Z = g0,g2 ǐ

#4 $\check{u}_x\check{u}_y$ Klein-4



same

G1p1 — $N_{BasEl}=2$



- **g0 #1** : s >0 scalar
- **g1 #2** : P_0 ⋮ P_1 or $\vec{e}_x \equiv P_1 - P_0$; carries: $P_0 + \vec{e}_x = P_1$
- **g3 #1** : $\hat{I}_2 = P_0\wedge\vec{e}_x$

#4 $\check{u}_0\check{u}_x$

Boolean (Sets)

G0p1 — ǔA

A = { 0 No / Out / Tails ; 1 Yes / In / Heads } Dual

$\check{u} = \sqrt[2]{1} = \{1, -1\}$ #2

G0pm — ǔA, ǔB

(A, B ... independent) ...(m)
[no geometric carries]

Not ∧ or ∘ and
DeMorgan's

$\overline{A \wedge B} = \overline{A} \circ \overline{B}$
$\overline{A \circ B} = \overline{A} \wedge \overline{B}$

$\#2^m$

Maxwell / Lorentz Equations for Charge and "Spin"

Traditional 2D Fluid Algebras

$\mathcal{Cl}3$ Geometric Algebra — Clifford / Hestenes — 2D Tangent Space ⇔ Std. Vector Algebra — Gibbs / Heaviside — 1D+

	∘ Dot, ∧ Wedge		∘ Dot, × Cross
g0	$\nabla\circ\vec{E}$	⇔	$\nabla\circ\vec{E} = 4\pi(\rho_+ - \rho_-)$ (2-fluids)
g1	$-\nabla\circ\widehat{B}$	⇔	$\nabla\times\overleftarrow{B} = \frac{\partial}{\partial ct}\vec{E} + 4\pi\,\vec{J}/c$
g2	$\nabla\wedge\vec{E}$	⇔	$\nabla\times\vec{E} + \frac{\partial}{\partial ct}\overleftarrow{B} = 0$
g3	$\nabla\wedge\widehat{B}$ (bi-vec)	⇔	$\nabla\circ\overleftarrow{B} = 0$ (pseudo-vector)

Q

$+\mathcal{H}$ — $\mathcal{H}$elicity [Dirac MM]

Geometric Product

$A \,\mathring{\wedge}\, B \;\equiv\; A\wedge B + A\circ B$

∧ Orthogonal Extension

∘ Parallel Contraction

Cl3 and G3p1 build graded structures with the Geometric Product.
G3p1 builds up more "entanglement".

Units

$\mathbb{L}$, $\mathbb{T}$ — meter, (arb); c defines Time

$\mathcal{E}_0 \equiv m_e c^2$ (bound)

Length scales of E & M

$R_e = \dfrac{e^2}{\mathcal{E}_0} \sim 2.82 \times 10^{-15}$ m

$D_v = \dfrac{\hbar c}{\mathcal{E}_0} \sim 386. \times 10^{-15}$ m

Standard Vector "Algebra" becomes inconsistent at 1D+ pseudo-vector and can not represent 2 fluids .
Cl3 is a consistent Linear Algebra, with 3 Basis Vectors, giving for a 2D Tangent Space, but cannot represent Particles.

PC$\mathcal{H}$ Symmety

$\check{u} = \{+, -\}$

Geometric Parity $\check{u}_0\ \check{u}_x\ \check{u}_y\ \check{u}_z$

	Bound Particles			
	e−	p+	e+	p−
Charge $\check{u}_Q$	−	+	+	−
$\mathcal{H}$elicity $\check{u}_H$	+	+	−	−

G3p1 linear algebra connects 3-D Fields to Particle Sources with Charge and 3-D Helicity

1-Sided "Nascent" Potentiality

Nascent Fields — Source Points @(x_s,τ_s) — Charge — Moving Charge — Dipole Charge — Time Delay

dual

g0 $[\#e/\mathbb{L}]$: $\boldsymbol{\varphi}(x_f,t) = e\int d\mathring{x}\,\mathring{\wedge} \sum_s\left\{ \dfrac{\check{u}_s\mathring{P}_s}{R}\,\mathring{\wedge}\left(1 + \dfrac{\vec{\beta}\circ\vec{r}}{R} + \dfrac{\vec{d}\circ\vec{r}}{R}\right)\right\}^{@\tau_s = t - R_{sf}/c}$

$\vec{r}_{sf} = \mathring{P}_s - \mathring{P}_f$, $R = |r_{sf}|$

g1 $[\#e/\mathbb{L}^2]$: $\vec{E} \equiv -\vec{\nabla}\wedge\boldsymbol{\varphi}$

$\vec{\beta} = \frac{\partial}{\partial ct}(\mathring{P}_s - \mathring{P}_0)$

$J_{/\!/}$

$[\#e/\mathbb{L}^3]$:

g0: $\dfrac{\vec{\nabla}}{4\pi}\circ\vec{E} = -\dfrac{\nabla^2\boldsymbol{\varphi}}{4\pi} = e\sum_s\left\{\check{u}_s\mathring{P}_s\,\mathring{\wedge}\left(1 + 2\beta_{/\!/} + 2d_{/\!/}\right)\right\}^{@\tau_s}$

if $\vec{d}_s \equiv P_s^{+} - P_s^{-} \neq 0$

g1: $\dfrac{\vec{\nabla}}{4\pi}\circ\widehat{B} - \dfrac{\partial}{\partial ct}\vec{E} = 0$ — $\cancel{+\nabla\circ\nabla\circ\mathcal{H}}=0$

PhoTino — static

g2: $\vec{\nabla}\wedge\vec{E} + \dfrac{\partial}{\partial ct}\widehat{B} = 0$ — $\cancel{+\nabla\wedge\nabla\wedge\boldsymbol{\varphi}}=0$

$\widehat{M} = \check{u}_H\left(\dfrac{D_v}{2}\right)(1-\beta^2)(\vec{\beta}\circ\overset{\text{u}}{\widetilde{T}})$ Spin — i.e. $\widehat{M} \perp \vec{\beta}$

g3: $\vec{\nabla}\wedge\widehat{B} = \dfrac{\nabla^2\widetilde{\mathcal{H}}}{4\pi} = e\sum_s\left\{\check{u}_s\mathring{P}_s\,\mathring{\wedge}\left(0 + 2\vec{\beta}\wedge\vec{r}\circ\overset{\text{u}}{\widetilde{T}} + 2\vec{\beta}\wedge\widehat{M}\right)\right\}^{@\tau_s}$

$J_\perp$

g2 $[\#e/\mathbb{L}^2]$: $\widehat{B} \equiv \dfrac{\vec{\nabla}}{4\pi}\circ\widetilde{\mathcal{H}}$

Moving Charge — Bound Helicity

g3 $[\#e/\mathbb{L}]$: $\widetilde{\mathcal{H}}(x_f,t) = e\int d\mathring{x}\,\mathring{\wedge}\sum_s\left\{\dfrac{\check{u}_s\mathring{P}_s}{R}\,\mathring{\wedge}\left(0 + \dfrac{\vec{\beta}\wedge\vec{r}\circ\overset{\text{u}}{\widetilde{T}}}{R} + \dfrac{\vec{\beta}\wedge\widehat{M}}{R}\right)\right\}^{@\tau_s}$

Helicity — orbit — spin

2-Sided Energetics

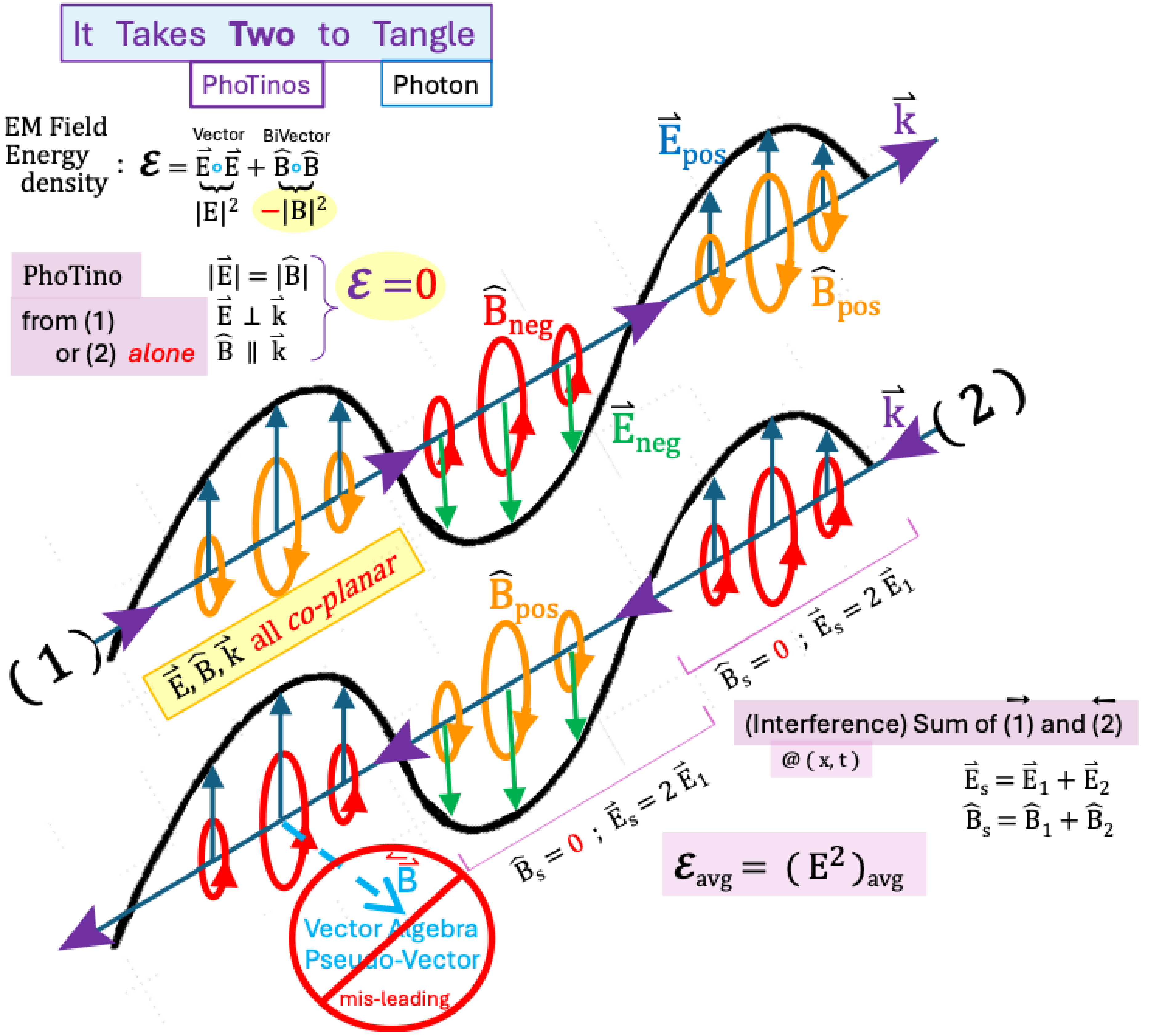
Energy is transferred between 2 Bound Particles by the symmetric Field / Particle interactions , "dual" between Charge / Helicity .

e e — e eD_v — e d e — eD_v eD_v

Energy: $\mathcal{E}\,\mathring{P} = q_f\mathring{P}_f\,\boldsymbol{\varphi}^{(x,t)} + e\mathring{P}\wedge\vec{d}\circ\vec{E}^{(x,t)} - q\mathring{P}\wedge\widehat{M}\circ\widehat{B}^{(x,t)}$

Force: $\vec{\mathcal{F}} = q\mathring{P}\wedge\vec{E}^{(x,t)} - q\mathring{P}\wedge\vec{\beta}\circ\widehat{B}^{(x,t)} + e\mathring{P}\wedge(\vec{d}\circ\vec{\nabla})\vec{E}^{(x,t)} - q\mathring{P}\wedge\widehat{M}\circ(\vec{\nabla}\circ\widehat{B}^{(x,t)})$

Torque: $\widehat{\tau} = + e\mathring{P}\wedge\vec{d}\wedge\vec{E}^{(x,t)} - q\mathring{P}\wedge\widehat{M}\,\mathring{\wedge}\,\widehat{B}^{(x,t)}$



Particles (1),(2) emanate "NilPotent" spherical waves w1,w2 ;
waves w1,w2 give simultaneous Forces on particles (1),(2) .