Geometric Electro-Magnetic Field Equations for Particles with Charge and Spin

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The complete and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) force and torque interactions between moving particles with charge and spin.
The algebra is the **4-level Grassmann linear algebra** describing 3D Euclidean space, with BasisElements of **{ 3 Vectors, 1 Point }**, generating structures for particles distinct from lines, planes, and volumes. Here, this is denoted , here denoted G3p1 .

With this full algebra, **Forces** from 2 particles can add to become a **Torque**, and two linear Flows can add to become a Circulation .
With similar clarity, the algebra describes the motional "transformations" between vector electric $\vec{E}$ and bi-vector magnetic $\hat{B}$ through 1st order (v/c) motion of an origin Point, without 2nd order space-time algebra effects.

Thus, the algebra explicitly distinguishes between conduction currents and the (orthogonal) spin/circulation currents;
and between dynamic effects such as spin-transfer Torque,
and entropic effects such as conduction Resistance or magnetization damping.

Geometrically, there are two fundamental lengths in electro-magnetism :
the "classical radius of the electron $\mathbf{R_e} = e^2/m_ec^2 = 2.82$ pm (10^{-15} m),
which scales the electric interaction energy between *two* charges; and the "Compton wavelength", here denoted $\mathbf{D_v} = \hbar c/m_ec^2 = 386.$ pm,
which scales the magnetic dipole moment of a *single* electron Spin.

Significantly, a single propagating E-M wave necessarily linkes both together, with $|\vec{E}| = |\hat{B}|$ in magnitude. Moreover, as a bi-vector, $\hat{B}$ squares to *negative*, making $\vec{E}^2+\hat{B}^2 = 0$; and this agrees with the first Lorentz invariant", generally written as $\vec{E}^2 - \vec{B}^2 = 0$.
Thus, a single E-M wave emanated from a particle does not by itself carry energy; but rather *two oppositely propagating waves* are required to transfer energy from one particle to another. This is closely analogous to the QM combination of $\Psi\Psi^*$.

Understanding these geometrical structures can substantially clarify the meaning of complex probability wave function analysis.

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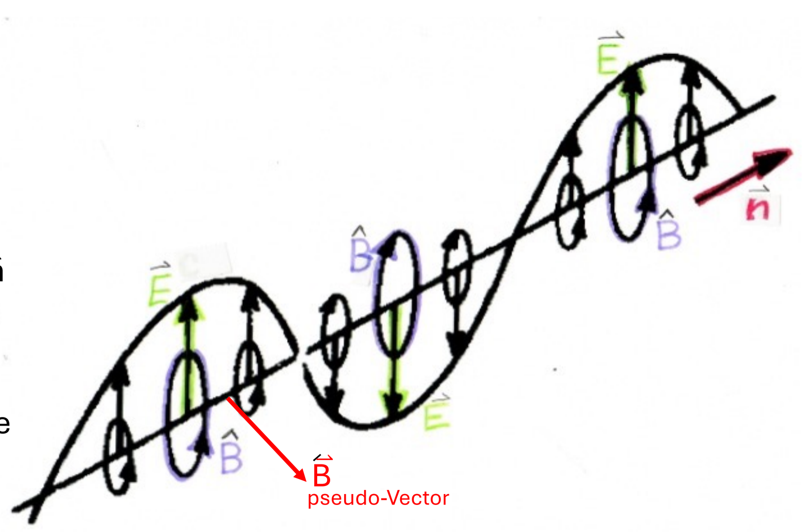
Travelling EM Waves from a single Particle (Charge & Spin) are NilPotent ; Interaction with 2nd Particle transfers Force & Torque .

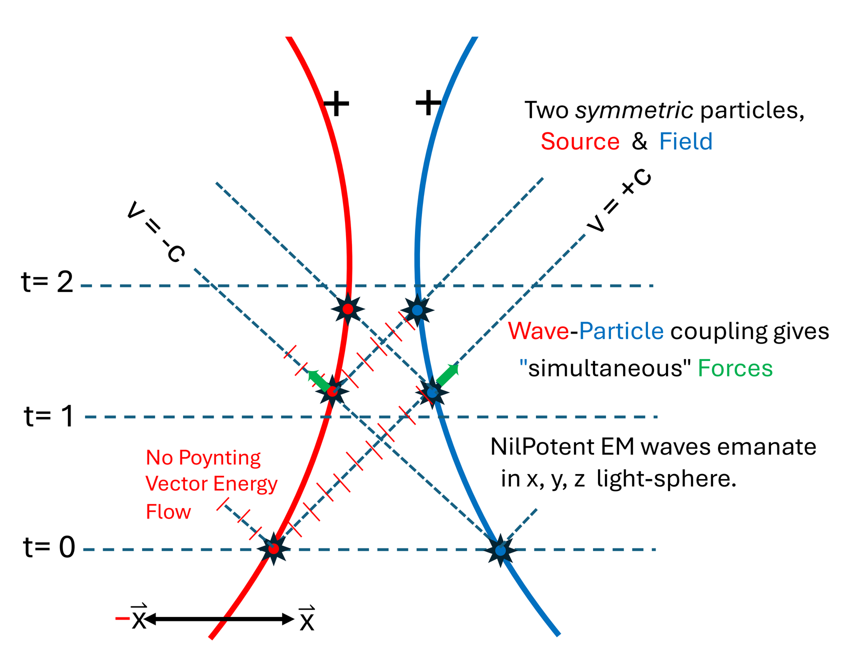
Always "Transverse"

with $|\vec{E}| = |\hat{B}|$
and $\vec{E} \perp \vec{n}$ and $\hat{B} \parallel \vec{n}$

NilPotent Travelling Wave

when $\vec{E} \parallel \hat{B}$
i.e $\vec{E} \perp \vec{B}$





G3p1 linear algebra connects **Fields** to **Particle Scources**

$\mathcal{C}\ell 3$ Fluid Maxwell

$$\nabla\circ\vec{E} = 4\pi(\rho_+ - \rho_-)$$

$$\nabla\wedge\hat{B} = \nabla\circ\hat{B} = 0$$

$$\nabla\times\vec{E} + \frac{\partial}{\partial ct}\hat{B} = 0$$

$$-\nabla\circ\hat{B} = \nabla\times\vec{B} = 4\pi\vec{J}_{/c} + \frac{\partial}{\partial ct}\vec{E}$$

Length scales of E & M

$$R_e = \frac{e^2}{m_ec^2}$$

$$D_v = \frac{\hbar c}{m_ec^2}$$

c defines Time

G3p1

$$r_{sf} = P_s - P_f$$

fields / sources / charge / dipole

$$\nabla\circ\vec{E} = \int d^3x \left\{ q_s \frac{P_s(\tau)}{R_{sf}} \left(1 + \frac{d_{/\!/}}{R_{sf}}\right) \right\}_{\tau = t - R/c}$$

$$\nabla\times\vec{E} = -\frac{\partial}{\partial ct}\hat{B}$$

$$\nabla\circ\hat{B} = -\frac{\partial}{\partial ct}\vec{E}$$

$$\nabla\times\hat{B} = \int d^3x \left\{ q_s \frac{\hat{M}_B(\tau)}{R_{sf}} \right\}_{\tau = t - R/c}$$

source

$$\hat{M}_B = q_s \frac{D_v}{2} c\, \hat{S}$$

Magnetic Dipole

Energy

$$\mathcal{E} = q_f\,\varphi - \hat{M}_f \,\mathring{\wedge}\, \hat{B}$$

Force

$$\vec{F}_f = q_f (R_e) \left\{ \vec{E} - \vec{\beta}_f \circ \hat{B} + \hat{M}_f \circ \vec{A} \right\}$$

Torque

$$\hat{\boldsymbol{\tau}}_f = q_f (R_e) \left\{ \hat{M}_f \,\mathring{\wedge}\, (\vec{\beta}_f \wedge \vec{E} - \hat{B}) \right\}$$

Geometric Linear Algebras

$\mathcal{C}\ell m$ ≡ Clifford "Geometric Algebra"
m Basis Elements, all **Vectors** (Hestenes)
? SpcDim

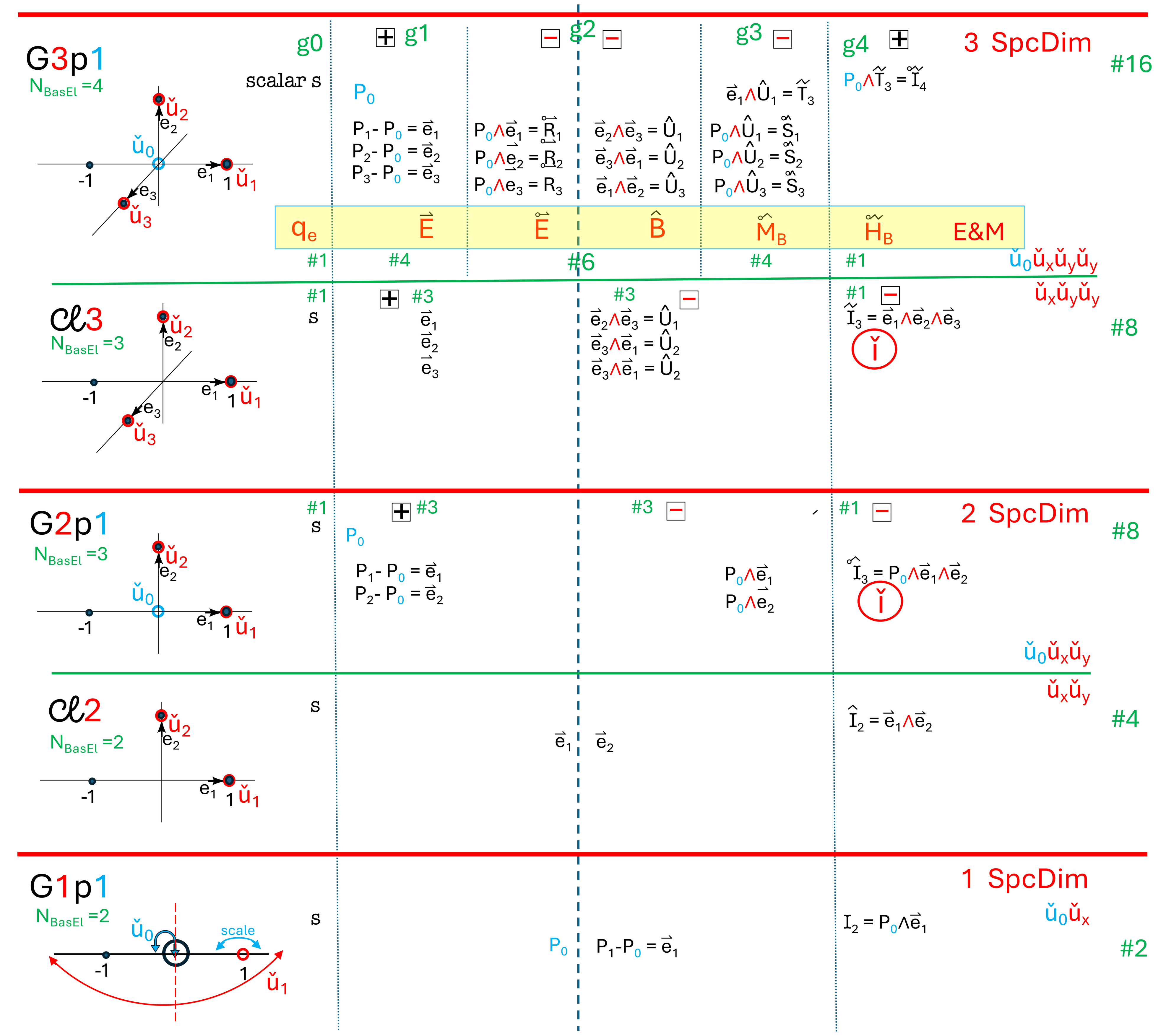
Geometric Product : $\vec{A}\diamond\vec{B} = \vec{A}\circ\vec{B} + \vec{A}\wedge\vec{B}$

Gnp1 ≡ Grassmann Extension Theory
n+1 Basis Elements : 1 **Point**, n **Vectors**
or : n+1 Points
n SpcDim

	$\mathring{P}$ Point	$\vec{e}_1 = \mathring{P}_1 - \mathring{P}_0$	FreVec
	$P\wedge e$ BndVec	$\hat{U} = \vec{e}_1\wedge\vec{e}_2$	FreBiVec circulation
spin	$P\wedge U$ BndBiVec	$\tilde{T} = \vec{e}_1\wedge\vec{e}_2\wedge\vec{e}_3$	FreTriVec
	$P\wedge T$ BndTriVec		

G3p1 gives full structures *needed* for geometric Particle/Field interactions in real 3-Space

$\check{u} = \sqrt[2]{1}$
$\check{\imath} = \sqrt[4]{1}$



Boolean (Sets)

G0p1
G0pm

A = { 0 No / Out / Tails ; 1 Yes / In / Heads } Dual

(independent)

#2

Not
Λ or
V and

$\overline{A \wedge B} = \overline{A} \vee \overline{B}$
$\overline{A \vee B} = \overline{A} \wedge \overline{B}$

#2m